Astrodynamics: Natural Orbits from Epicycles to Chaos

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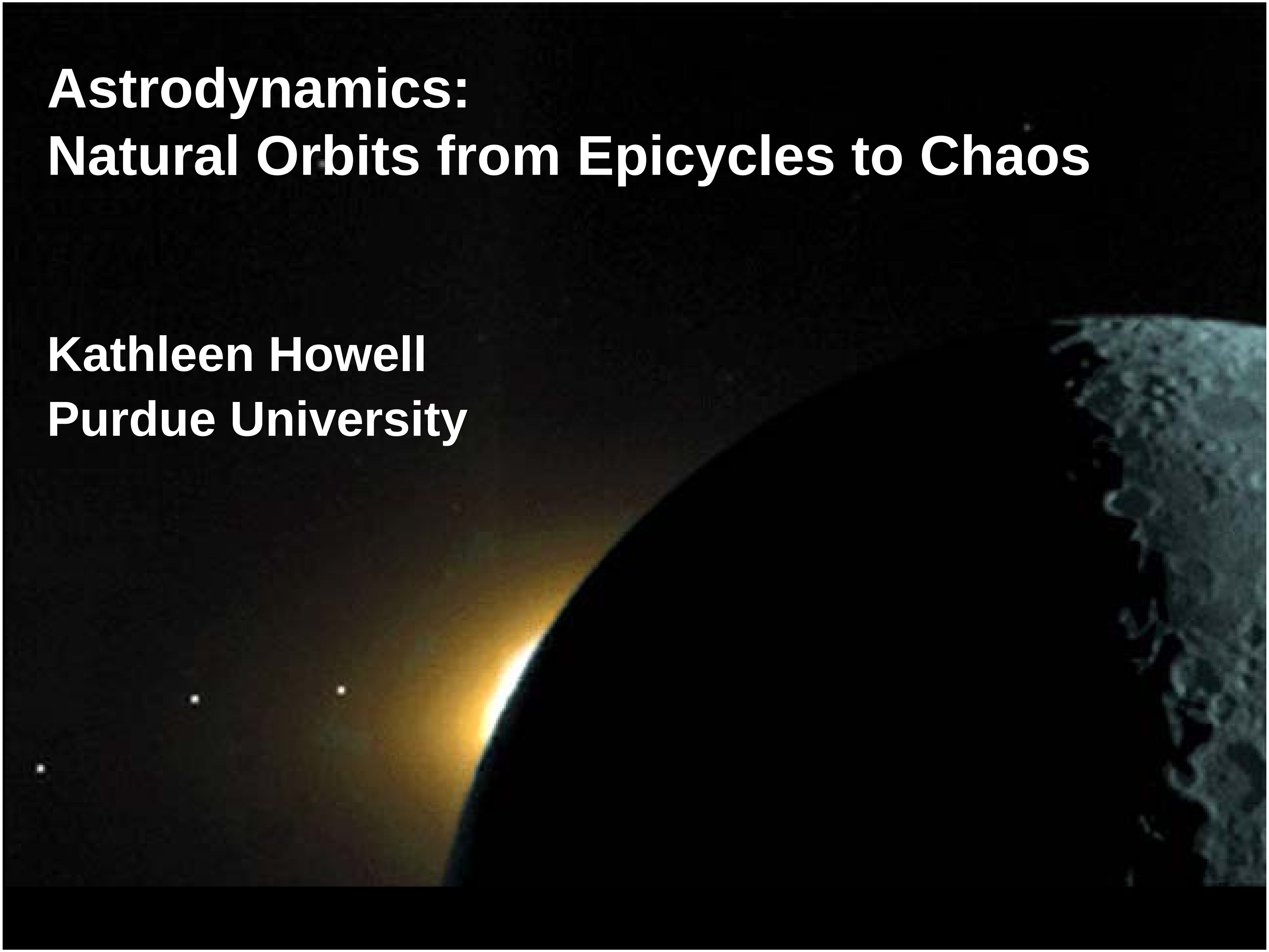
Purdue University



Introduction to Celestial Mechanics, Forest Moulton 1902

Celestial Mechanics and Astrodynamics

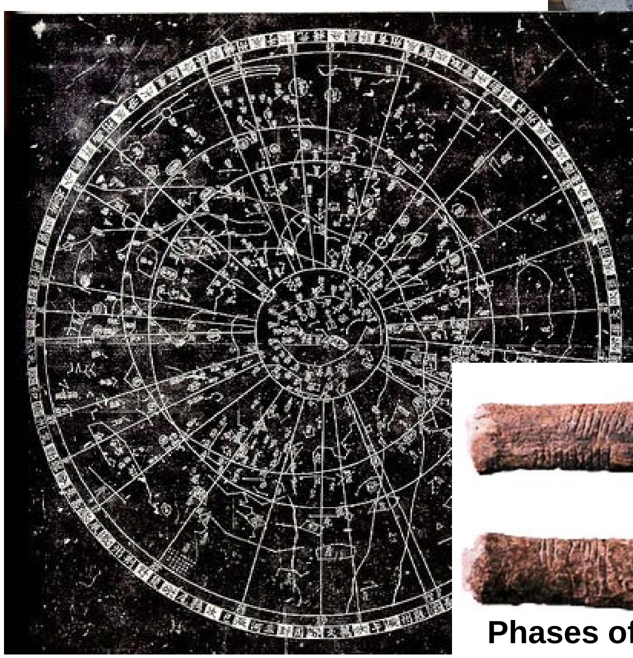
Formal Astronomy

Phenomena apart from causes:

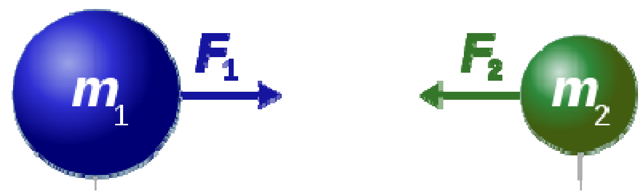
→ Divisions of time
→ Constellations
→ Planets

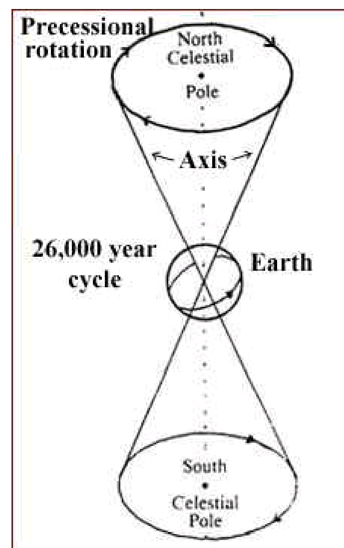
Dynamical Astronomy

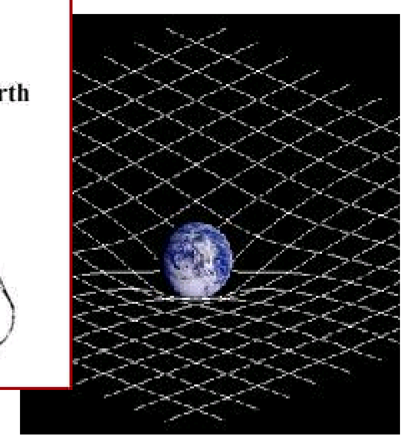
Physical aspects → natural phenomena
Fundamental properties → force, matter, space, time



Phases of Moon?





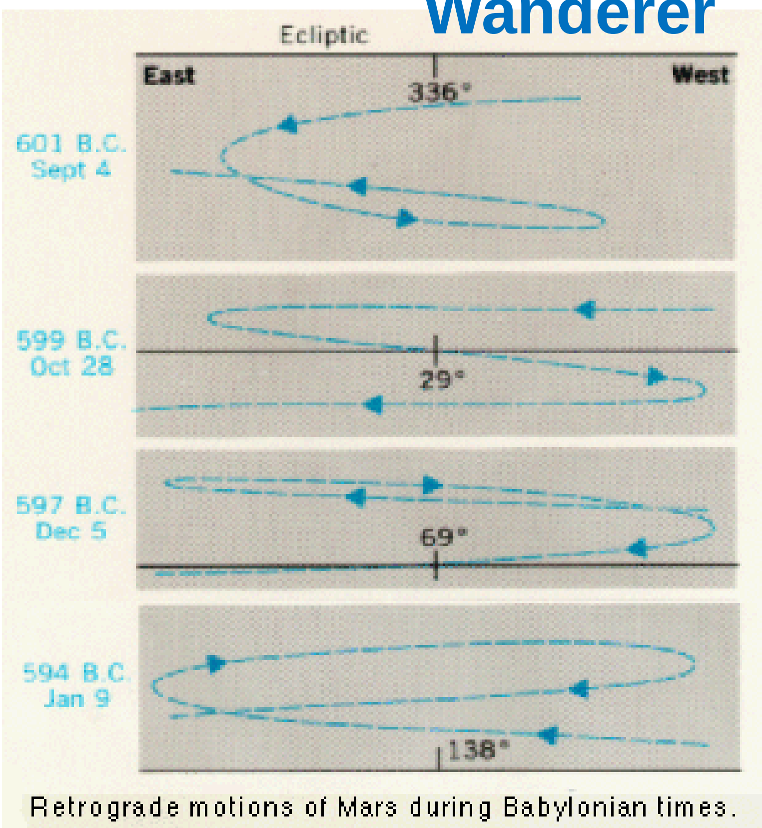


Ancient Astronomers

240 BC
Chinese astronomers → first confirmed perihelion passage of Halley's comet

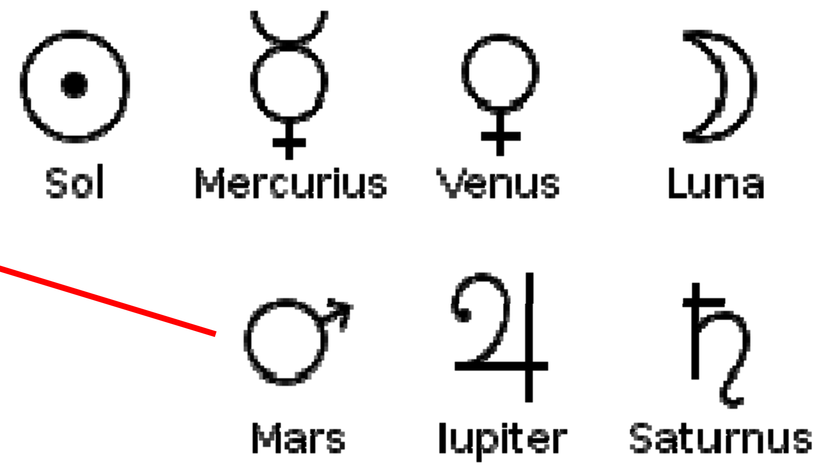
Ancient map of the stars – appear as flat screen circling world

Wanderer

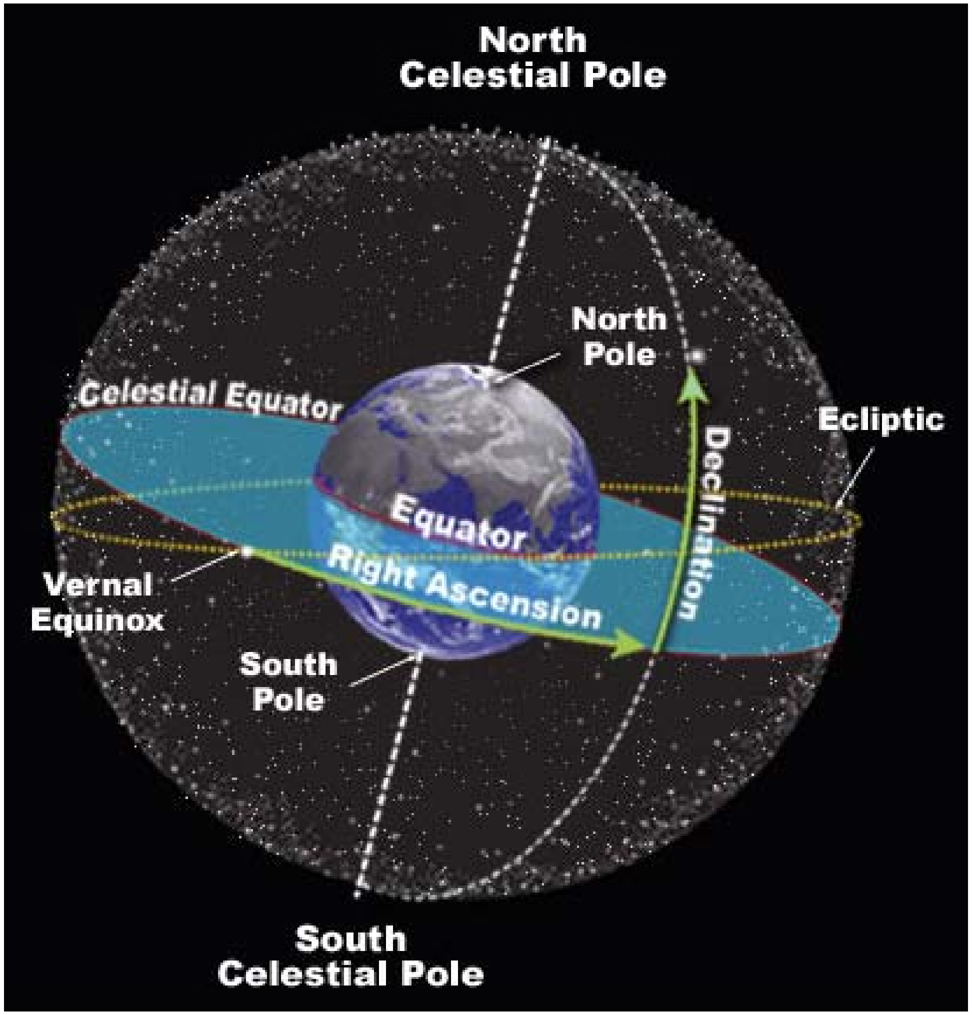


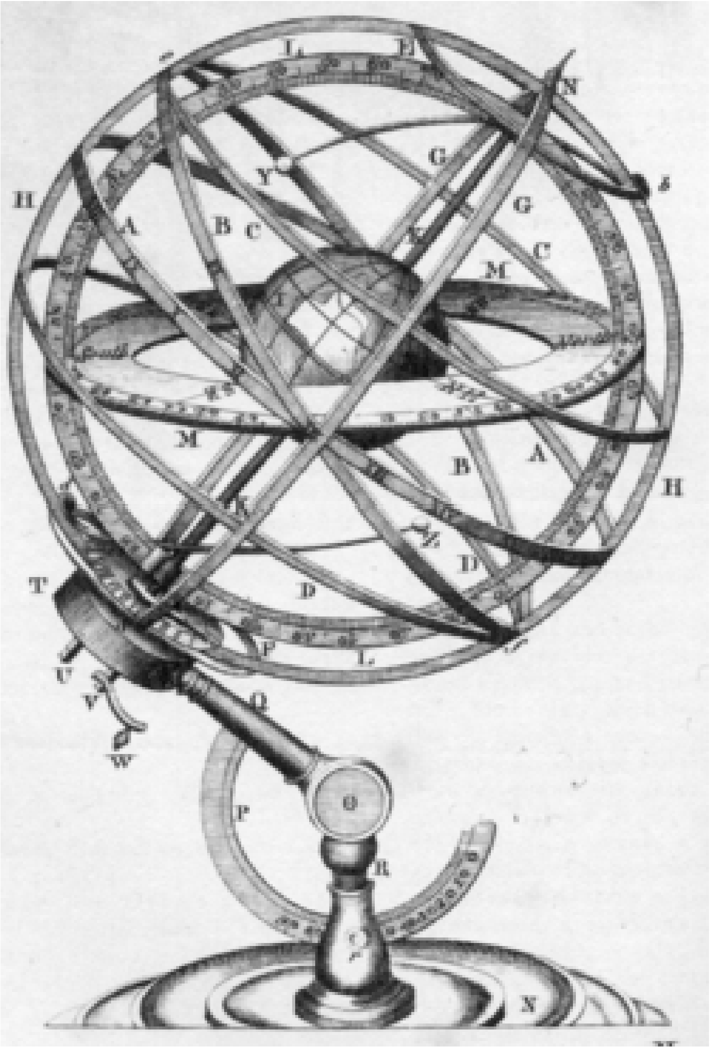
Retrograde motions of Mars during Babylonian times.

The 7 Planets of the Ancients



Celestial Sphere



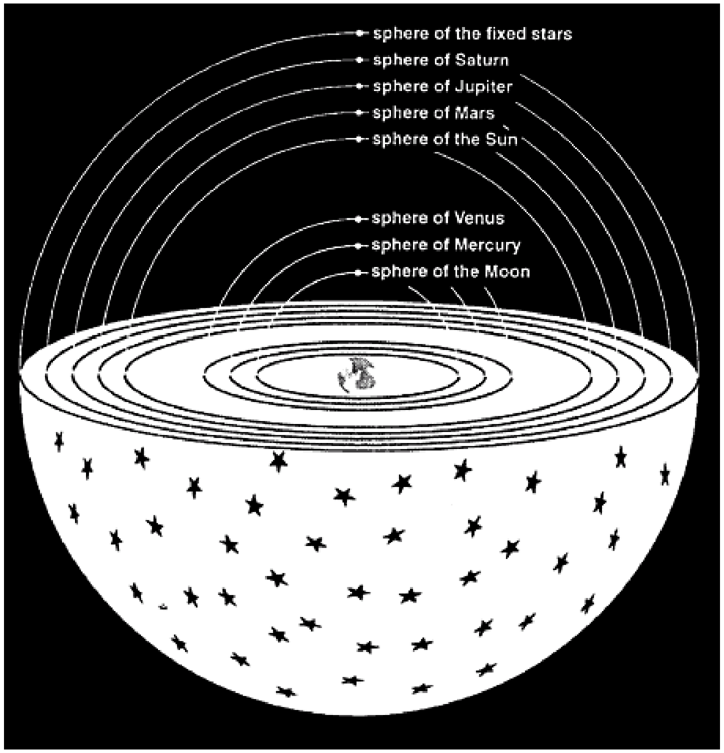


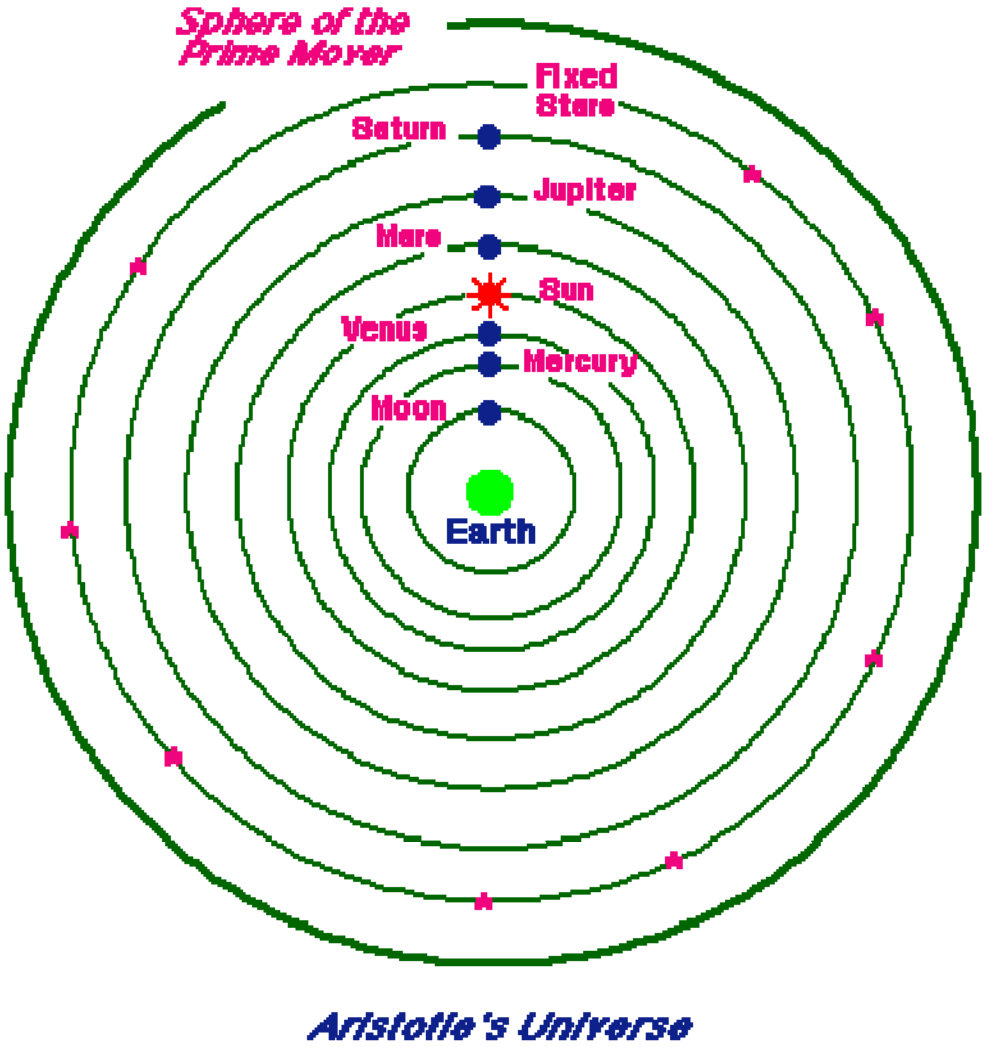
Imaginary sphere
- Arbitrarily large radius
- Concentric with Earth
- Rotates upon the same axis
- All objects projected upon celestial sphere

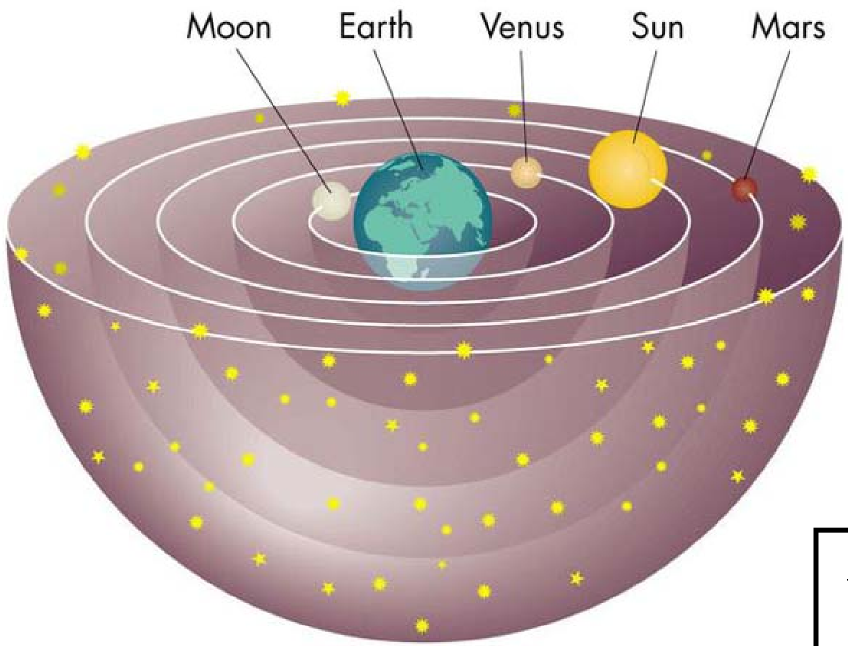
Aristole (384-322 BC)

Aristole’s Universe

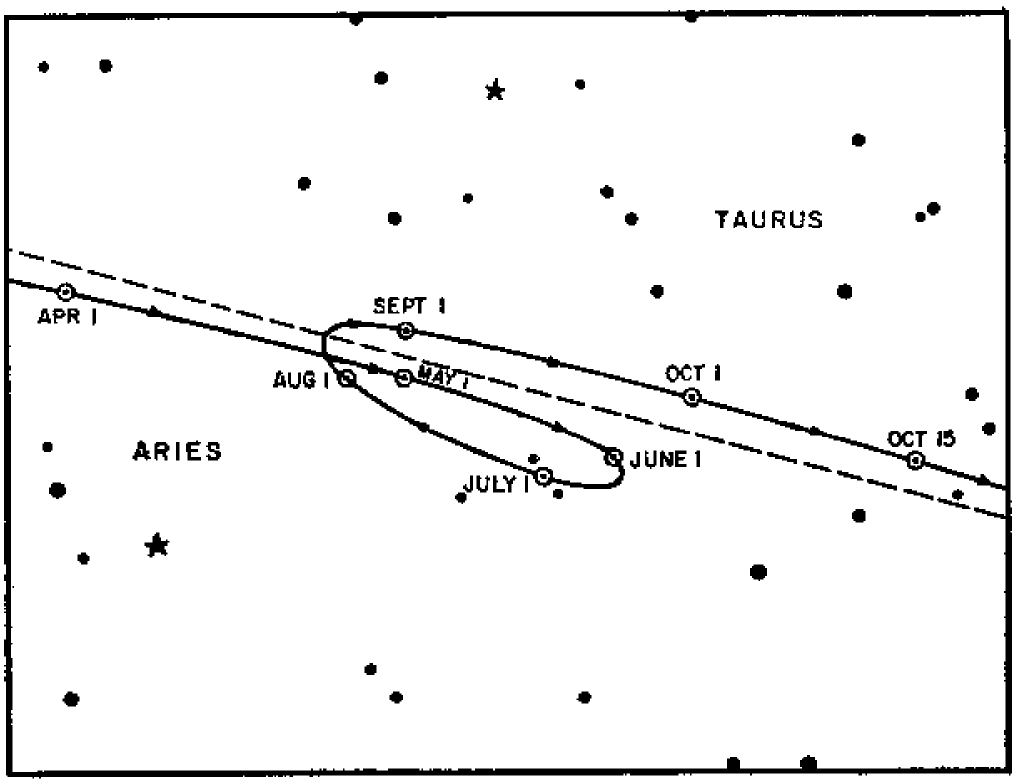
- 55 concentric, crystalline spheres
- Rotate at different velocities
- Angular velocity constant for given sphere
- Earth at center



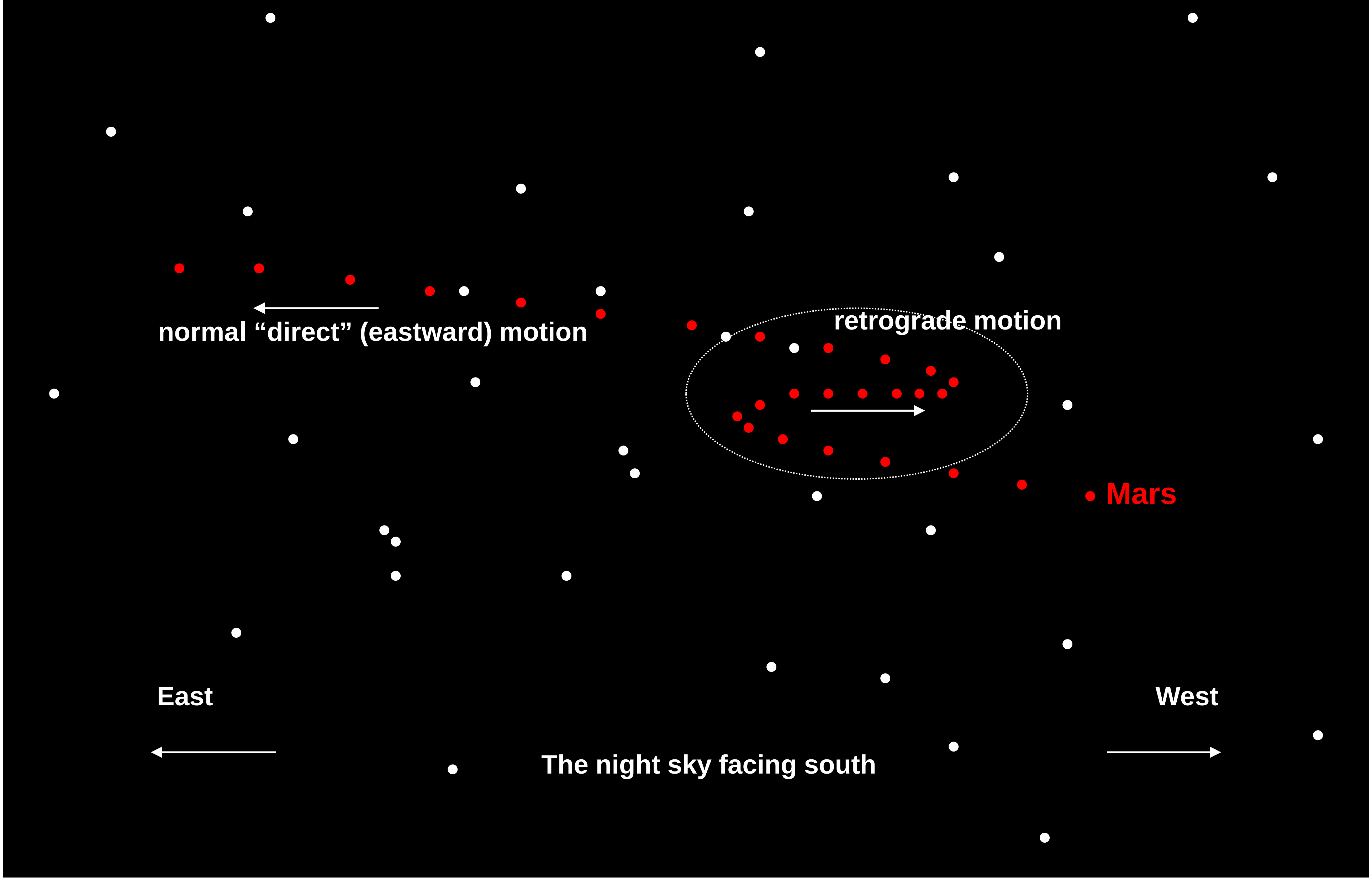




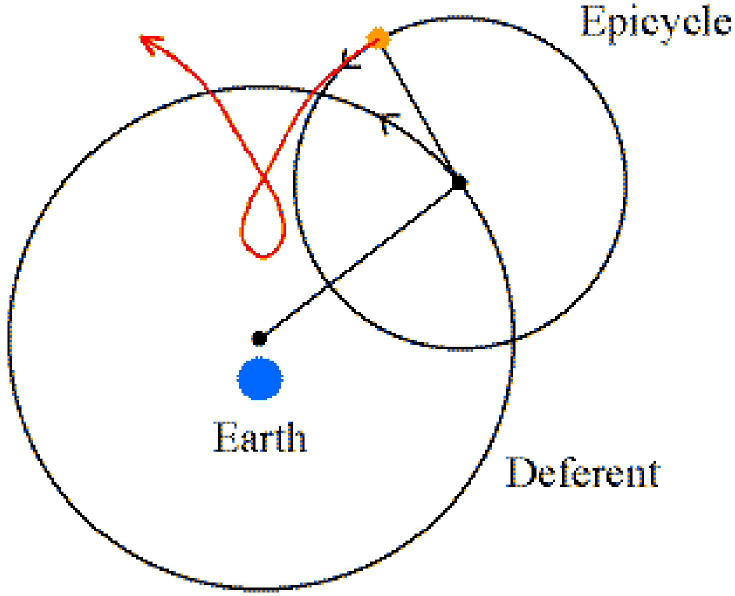
Concept does NOT produce observed motion

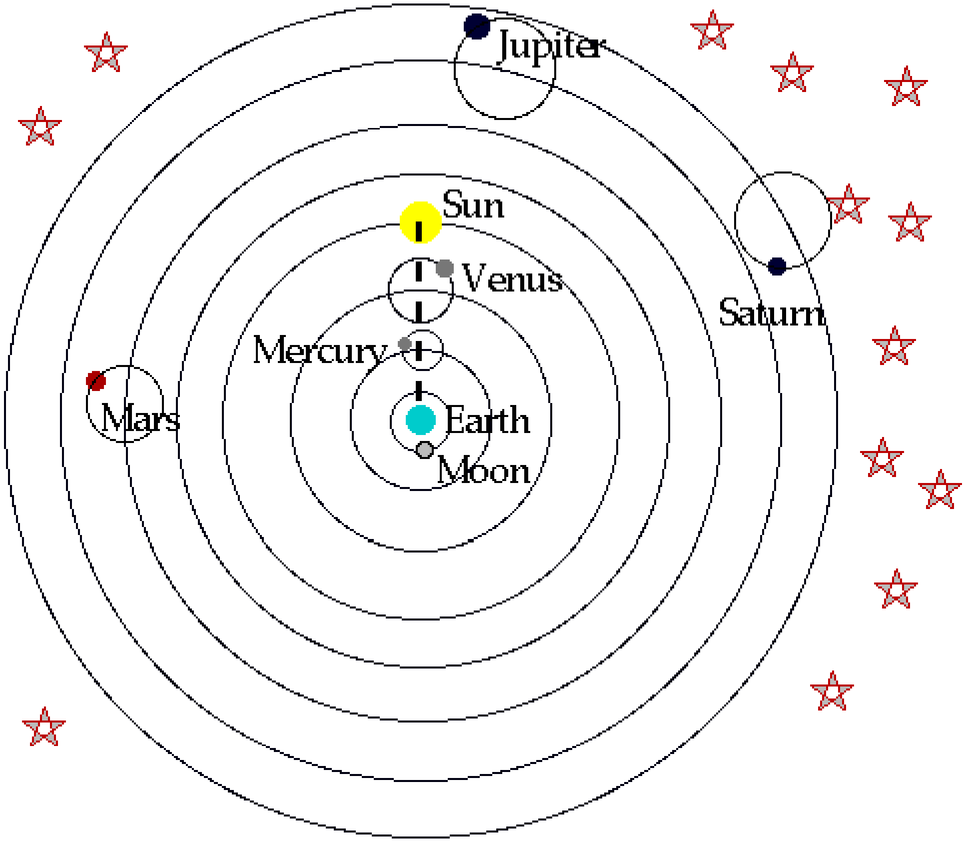


Motions of the “Wanderers” – The Planets



Epicycles



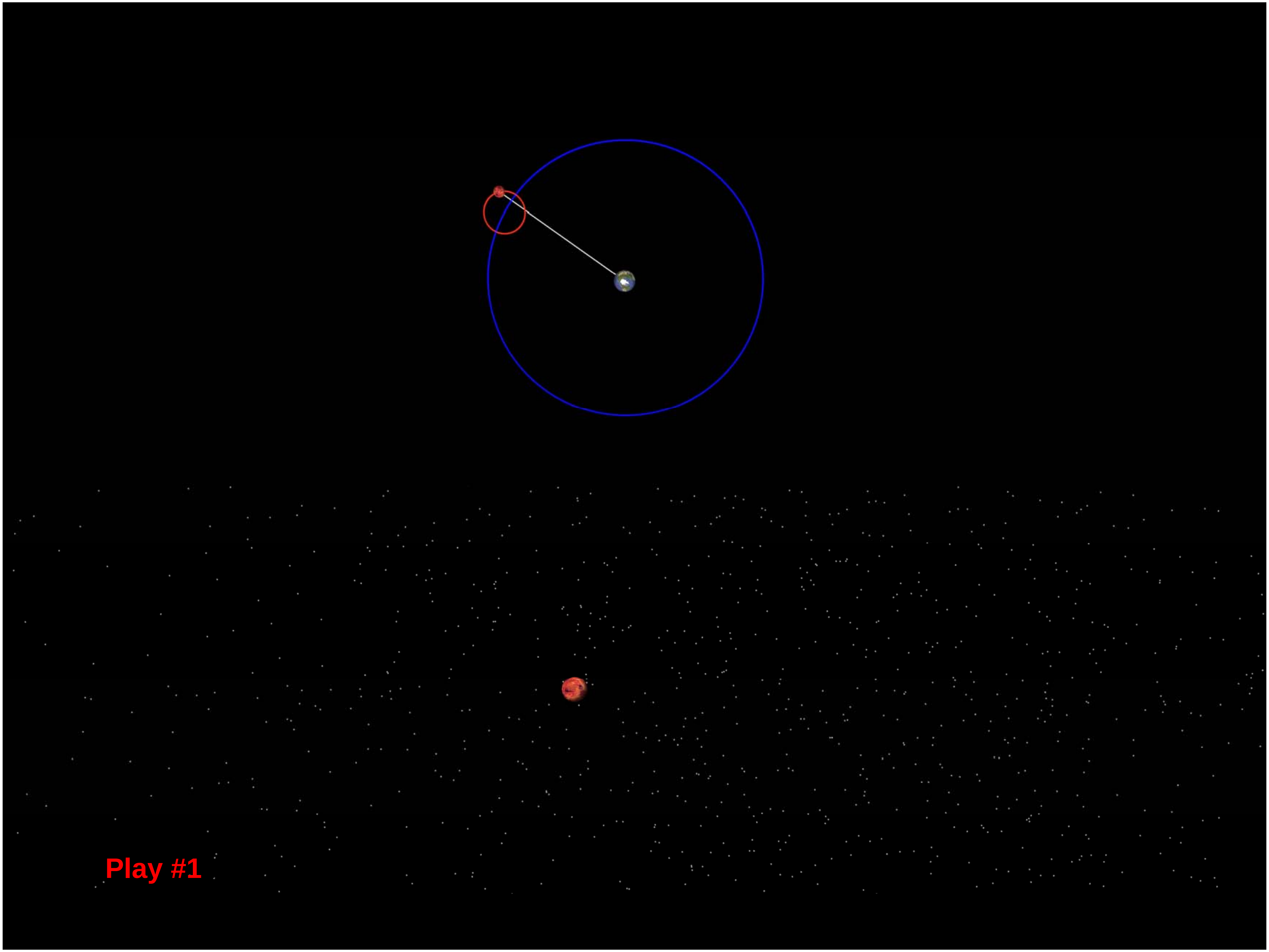


Planets → "Epicycles"

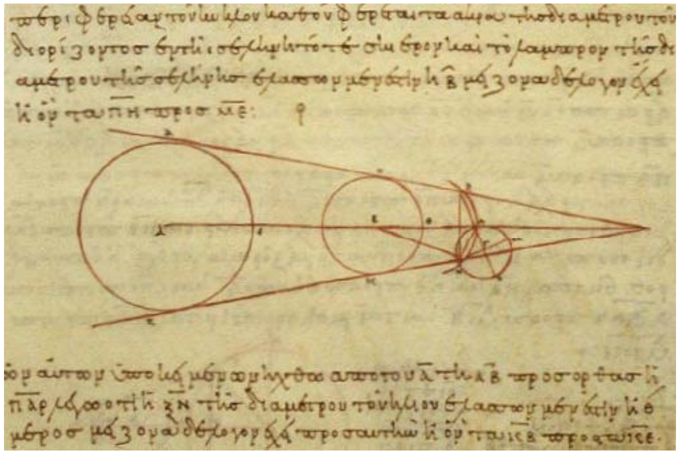
Concentric spheres → "Deferents"

Centers of epicycles → uniform circular motion

Epicylces → own uniform circular motion


Play #1

Heliocentric Theory?

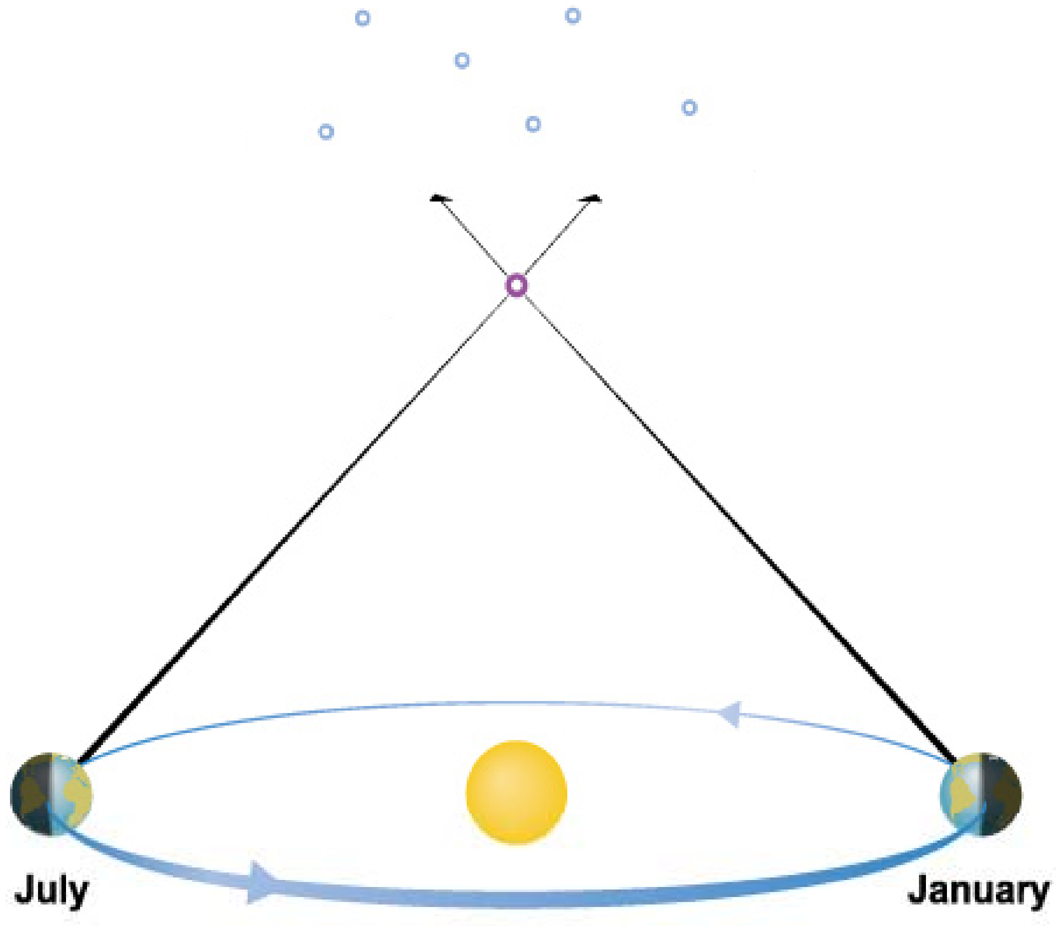


- Relative sizes of Sun, Earth, Moon
- Earth rotates in circle

Sun-Centered!

(no followers)

Aristarchus
(310-230 BC)



- Parallax – stars far away

- Planetary predictions poor

Earth-Centered!

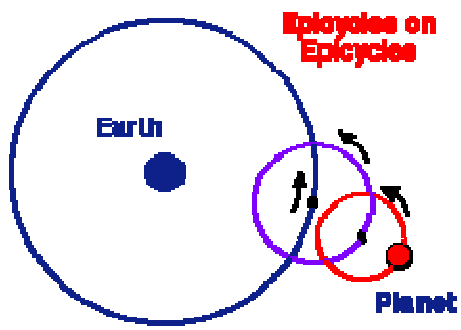
Hipparchus
(190-120 BC)

Ptolemy's Universe

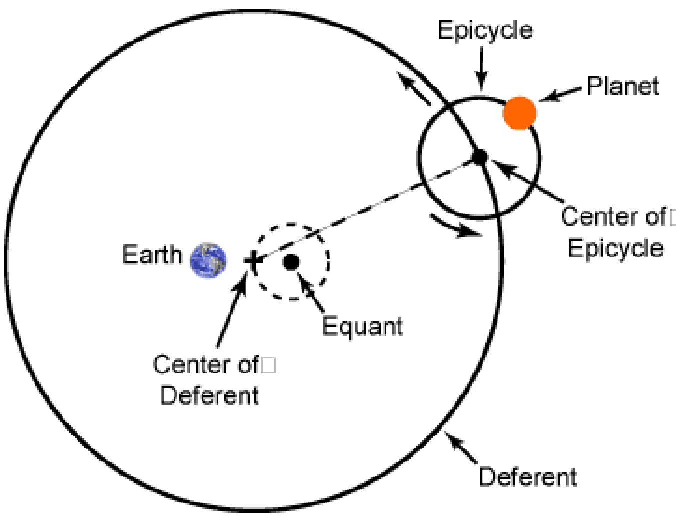
Required refinements:

→ epicycles on epicycles

→ center of the epicycle uniform motion about offset point



Claudius Ptolemy (100-170)



"uniform circular motion" :

1. All motion in the heavens → uniform circular
2. Objects in heavens from perfect material →cannot change intrinsic properties (e.g. brightness)
3. Earth at center of Universe
4. VERY GOOD predictions

Ideas catalogued by Ptolemy in Book:
"Almagest" (i.e., "The Greatest") 150 AD

→ "Ptolemaic Universe"

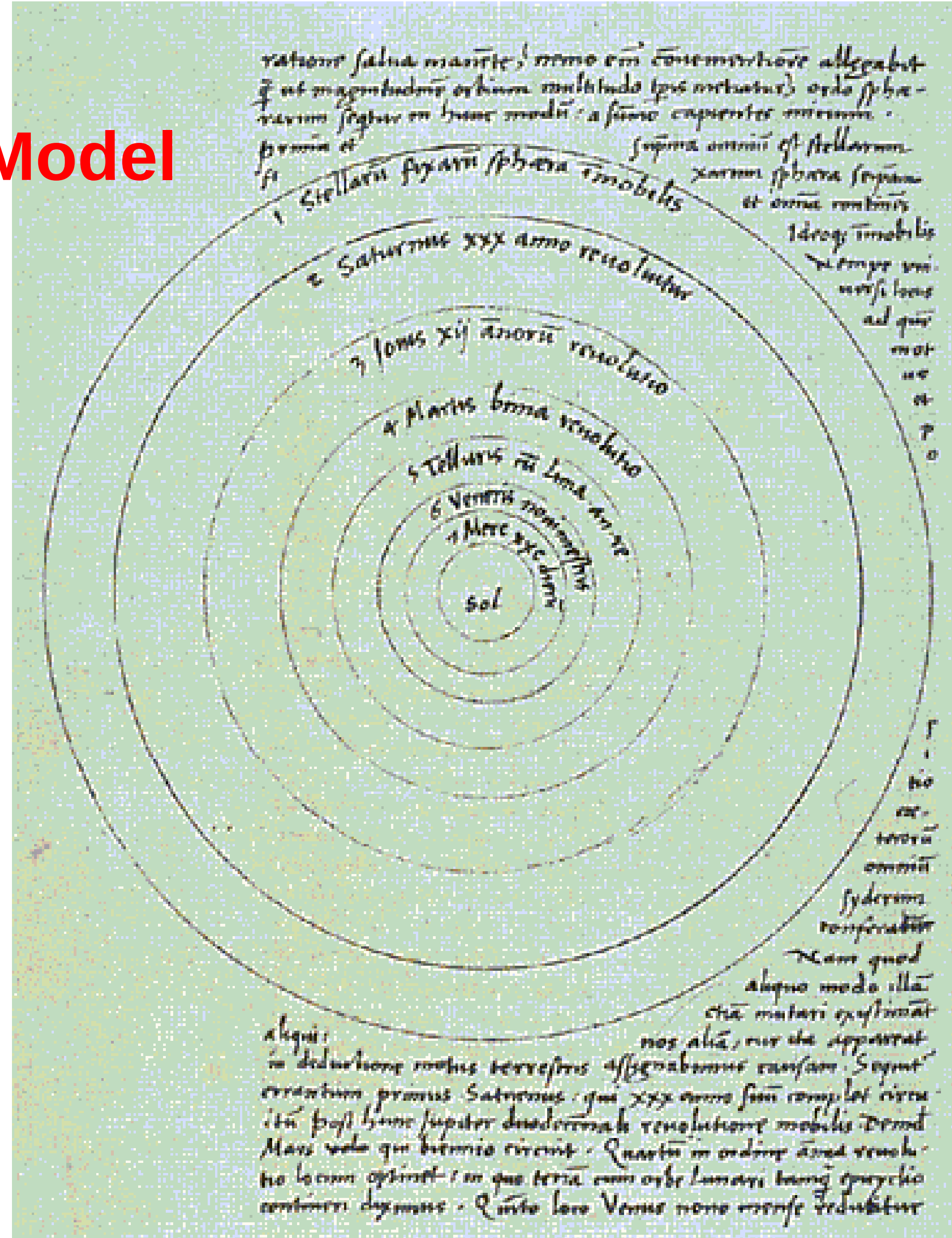
Copernicus: Heliocentric Model

Copernicus
(1473-1543)

Earth not fit to be center; Sun divine

Equant: betrayed concept of circles

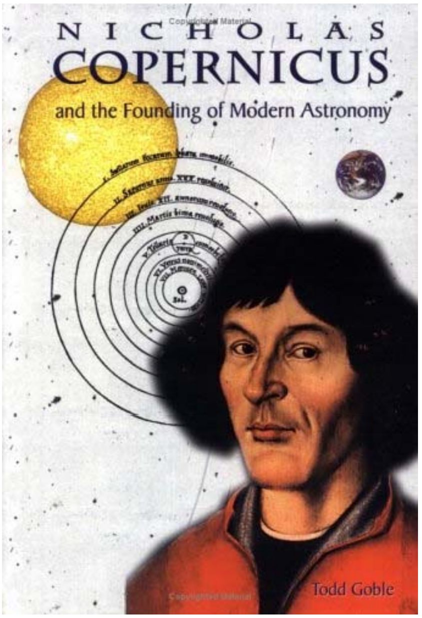
Sun + Epicycles → no equant



Copernicus' Model:

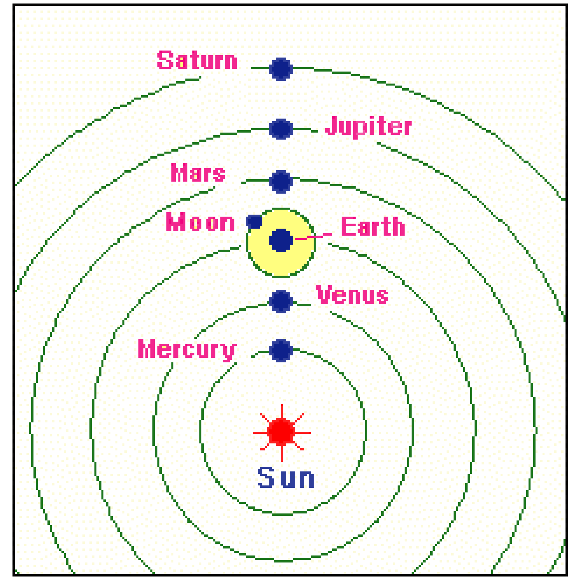
- No better results than Ptolemy
- Basic Info:
  - Sun at center
  - Stars far away
  - Earth rotates on axis
  - Earth rotates about Sun
- Simple Orbits

Still Required 48 epicycles!!

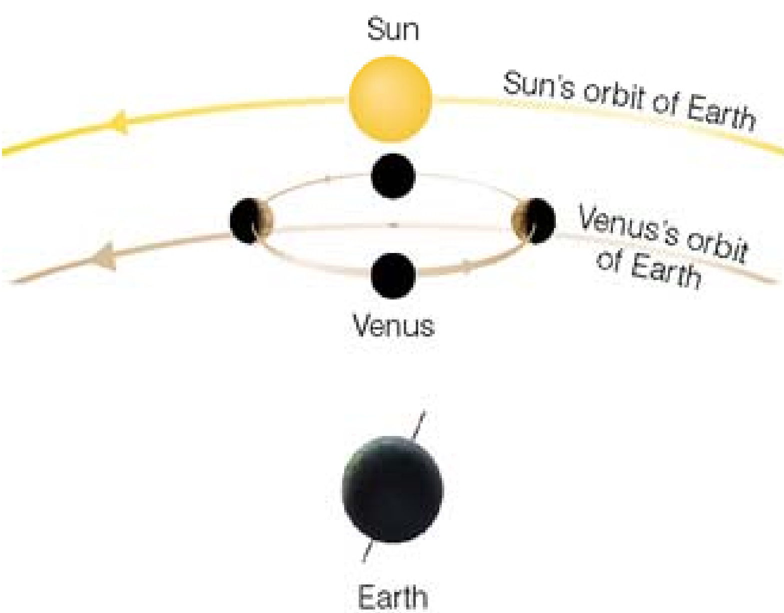


Copernicus' Universe

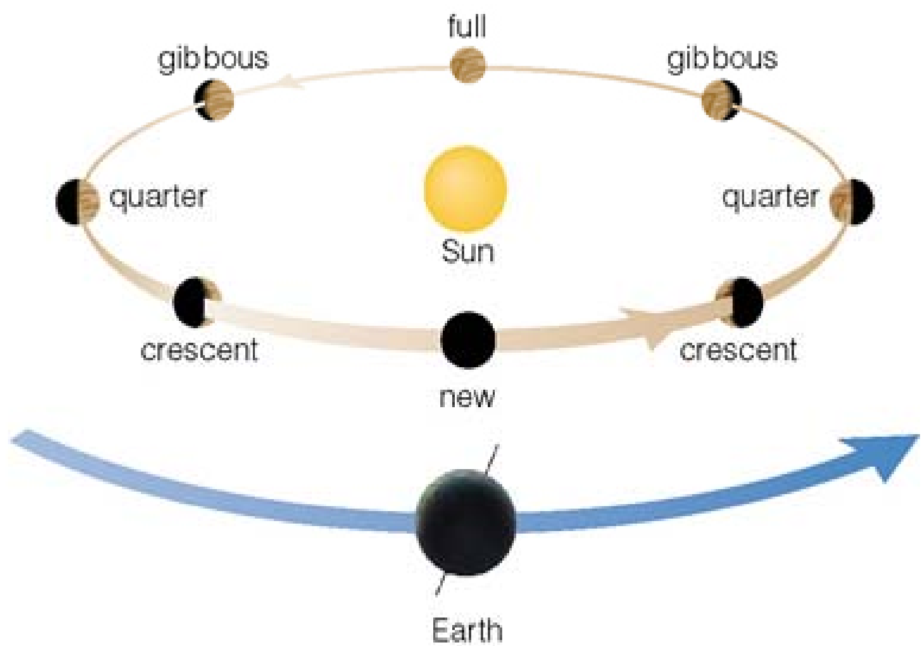
What combination of circles?



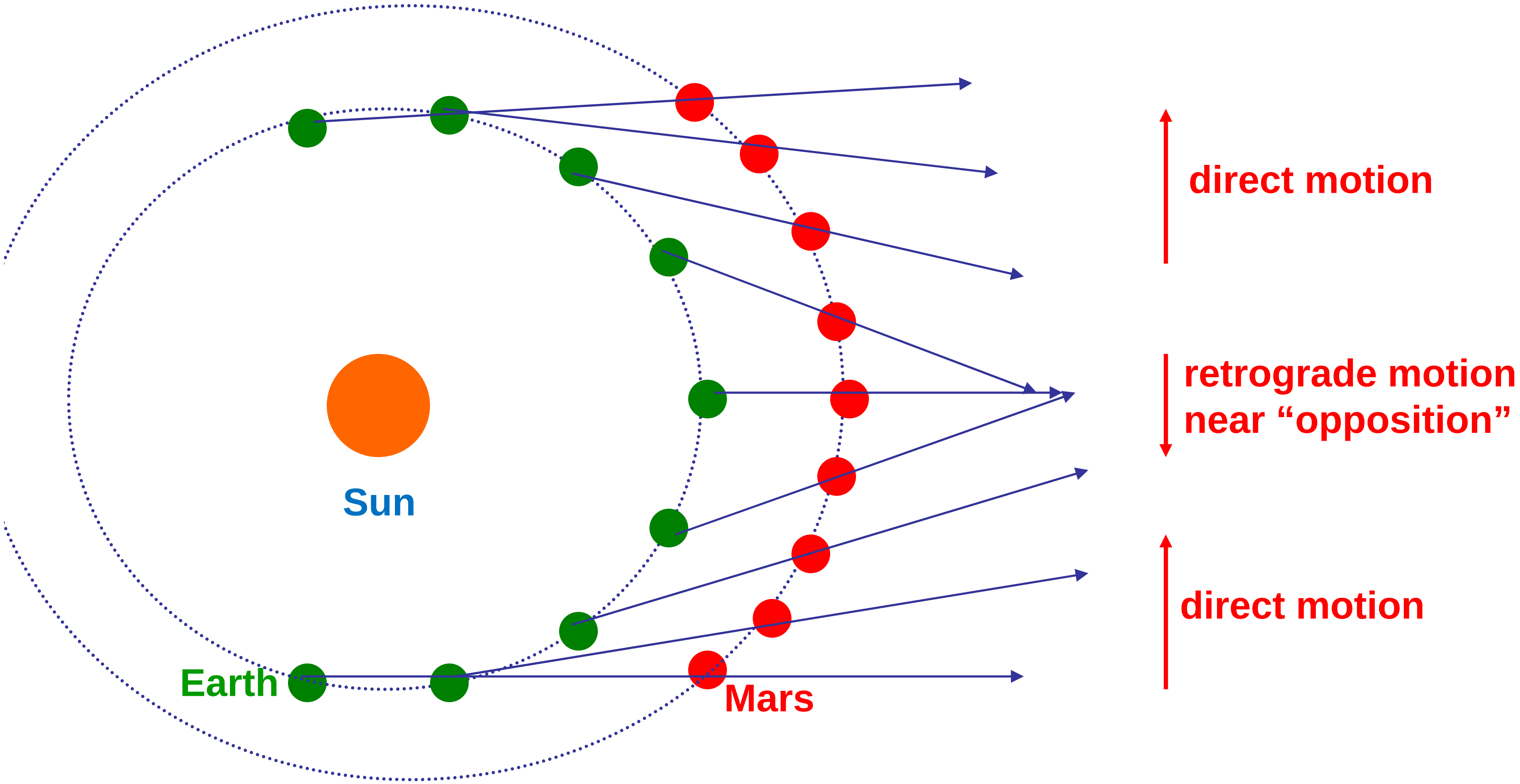
Ptolemaic View of Venus



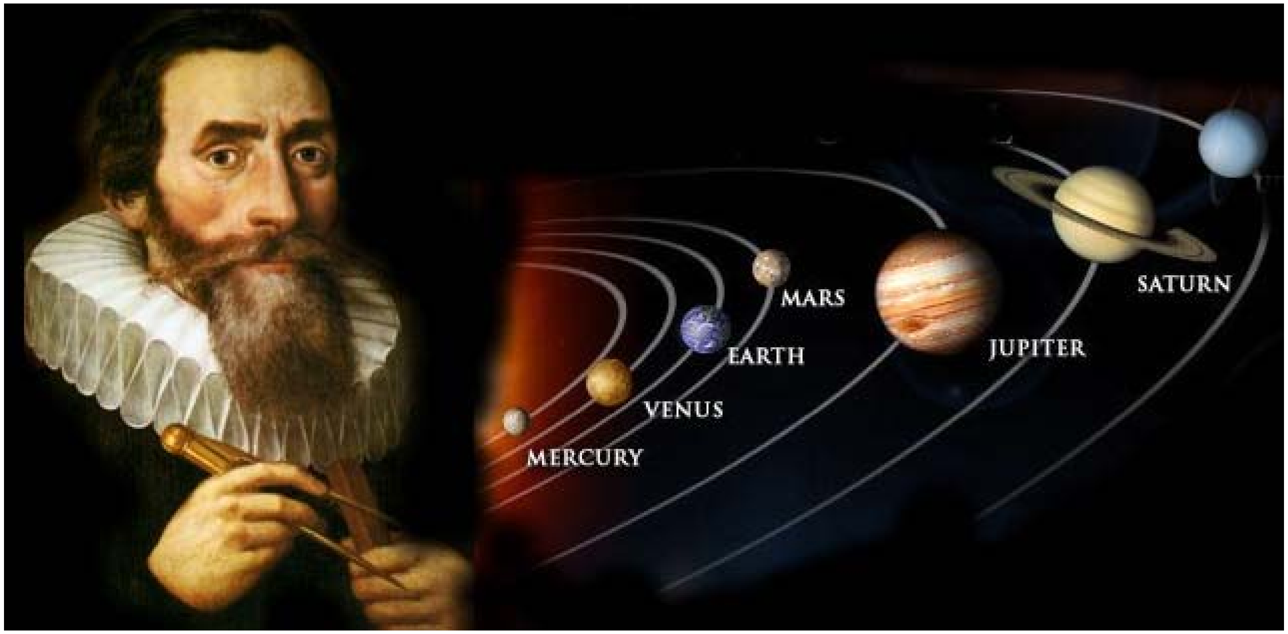
Copernican View of Venus



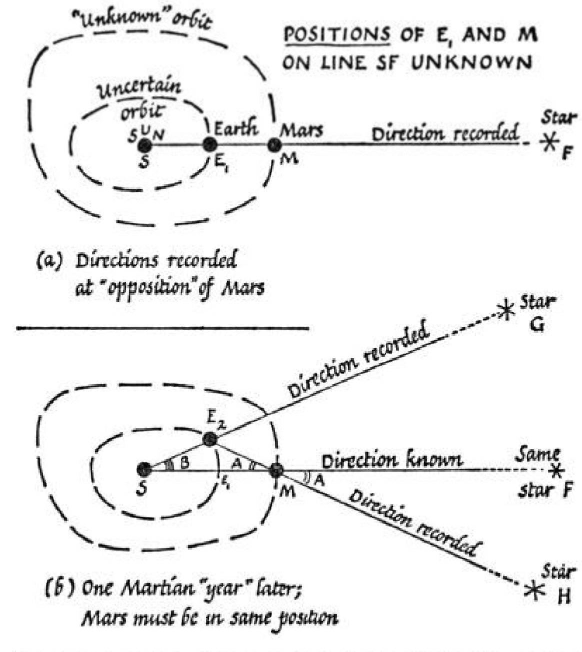
The Heleocentric Explanation



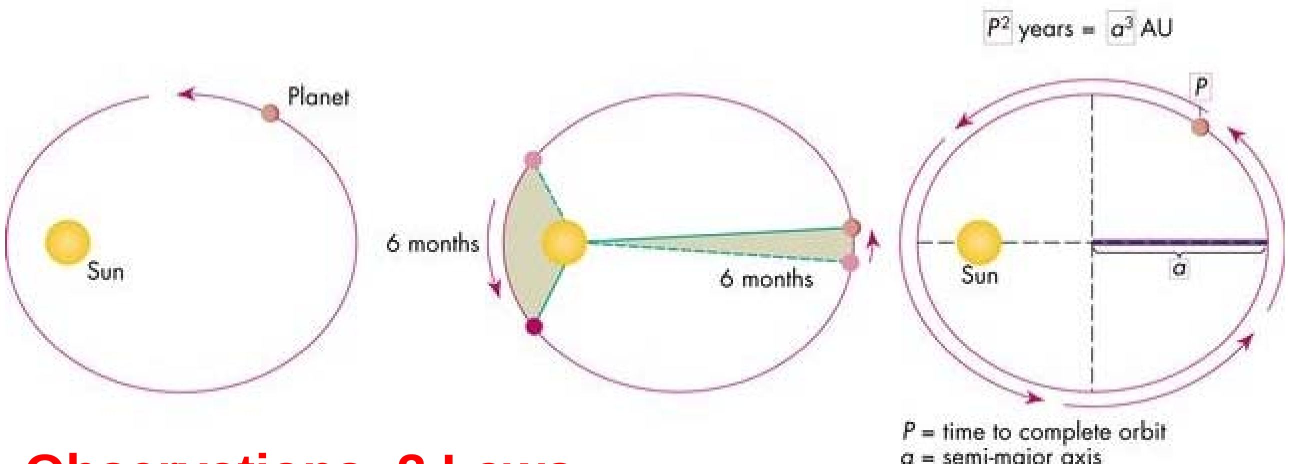
First proposed by Copernicus: ~1505
not published till *De Revolutionibus:* 1543

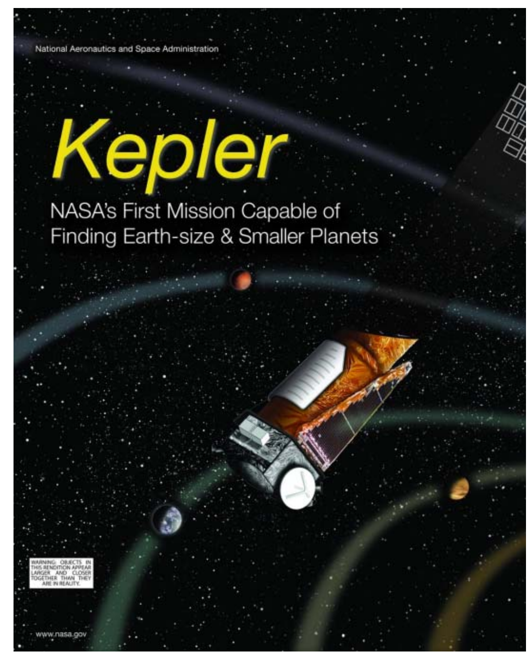


Johannes Kepler (1571-1630)



What do paths actually look like?

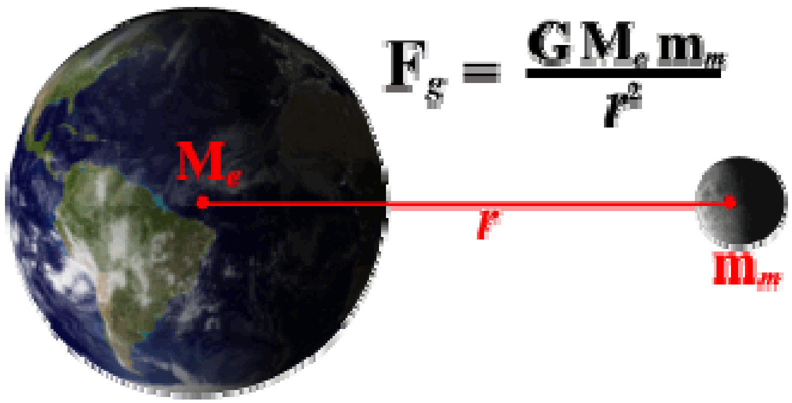




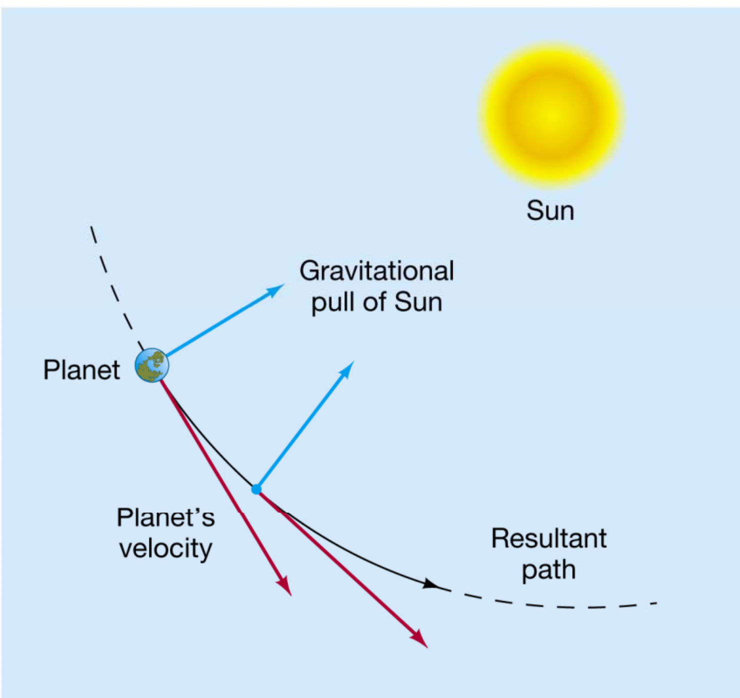
Observations: 3 Laws

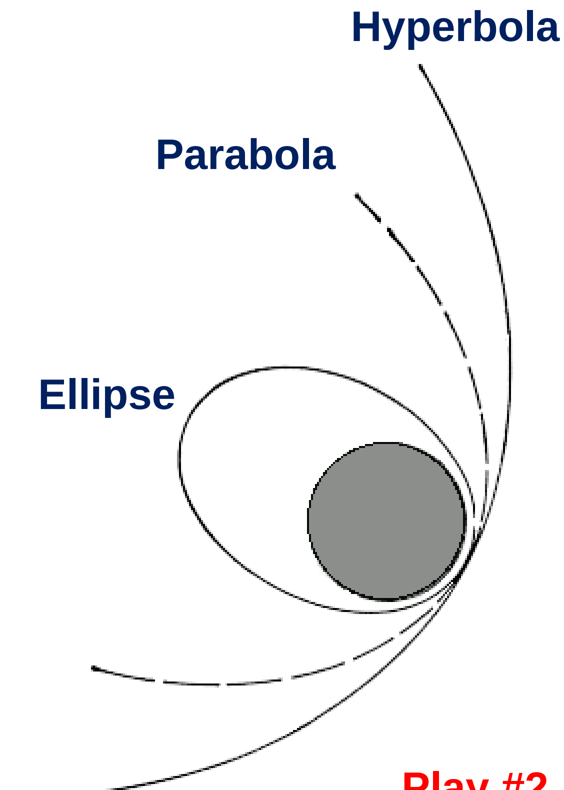
Newton → Law of Universal Gravitation

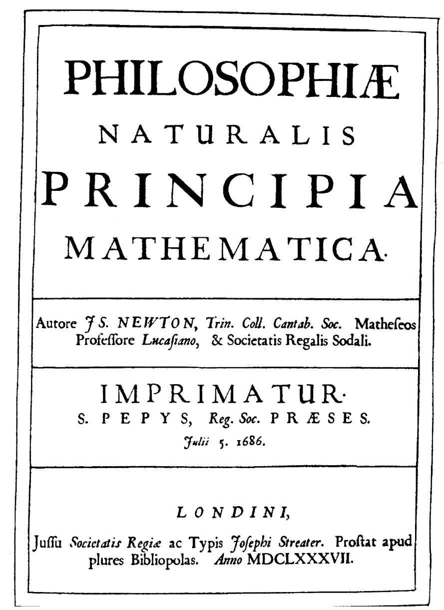
→ <u>Dynamics</u> of Celestial Bodies



Isaac Newton
(1687 –1725)







PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

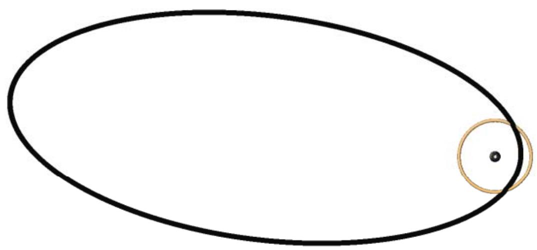
Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

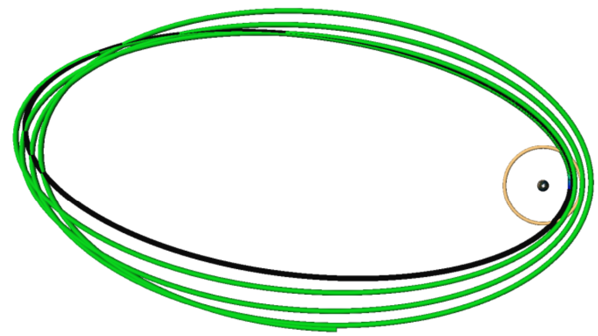
IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

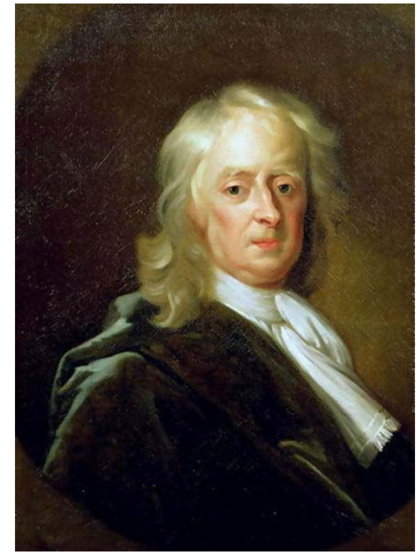
LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Play #2

Newton + Two-Body Problem







Gravitational interaction of 2 bodies ⇓ Planetary motion → 2BP	Deviation due to other bodies in solar system ⇓ Influence in principle via Newton's law

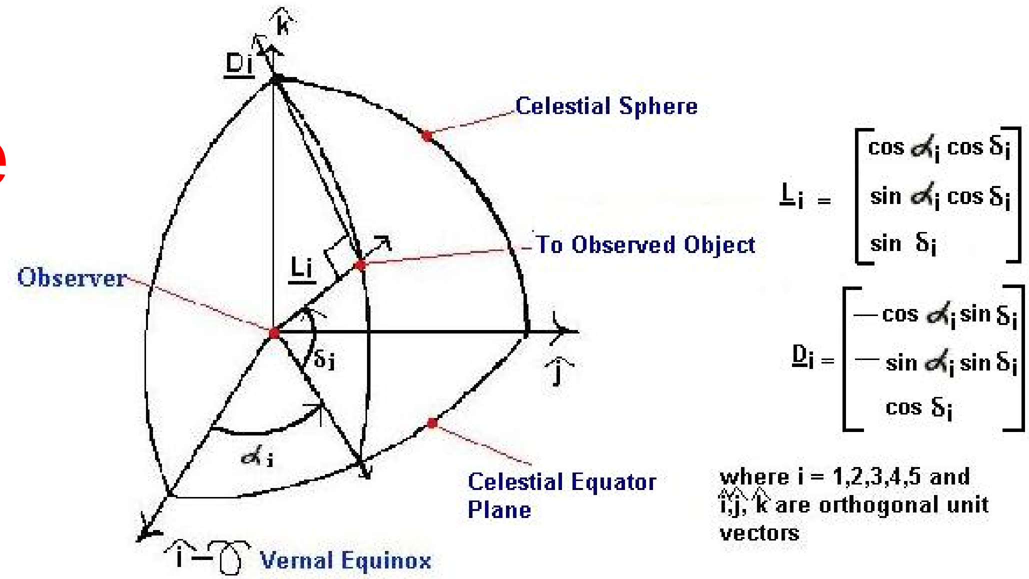
SUBSTANTIAL CHANGE

Pre-Newton:
- every orbit → combine circles
- correct because it works
- no basis: total solar system motion relies on mutual interactions

With Newton:
- each orbit can be exact
- incorporate ALL gravitational bodies ⟹ **N-Body Problem!**

Laplace (1749-1827)

Laplace Universe

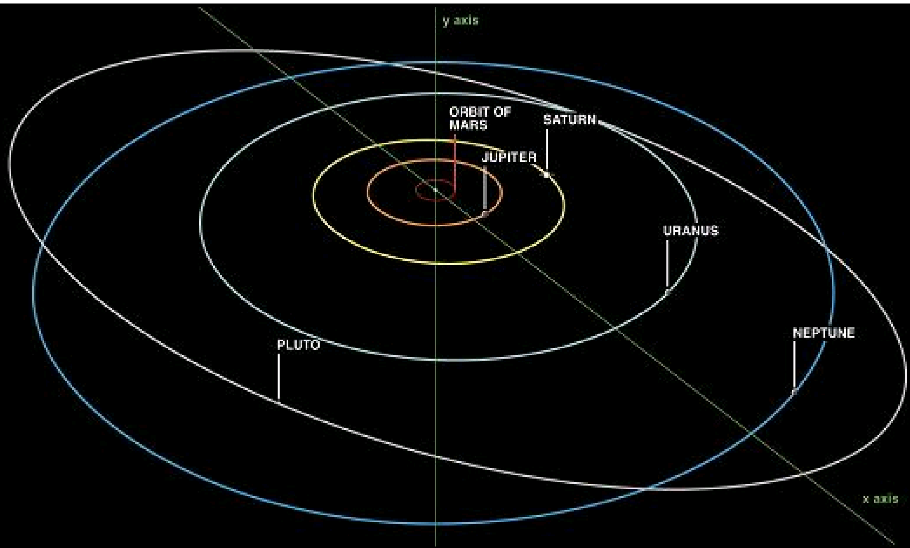


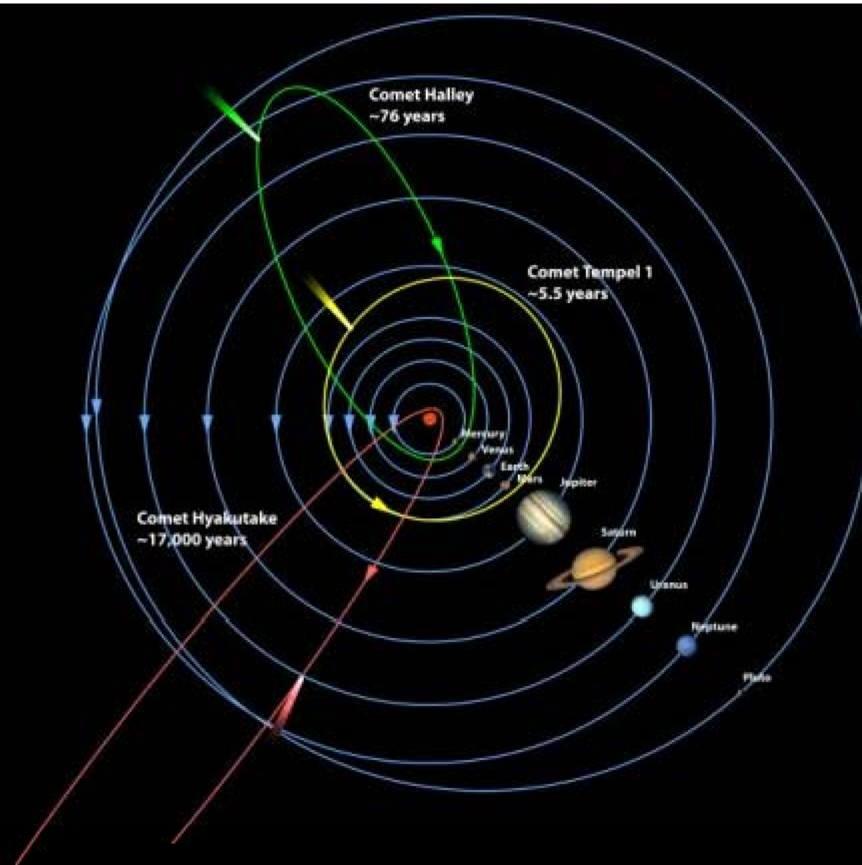
Universe is gigantic and perfect watch!

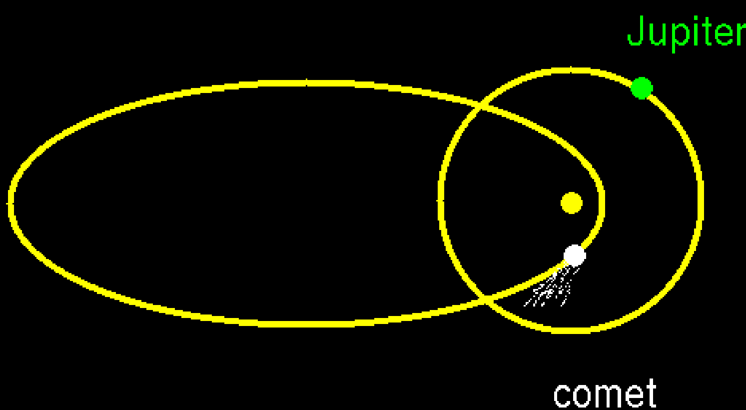
Problem = Conics + small disturbances

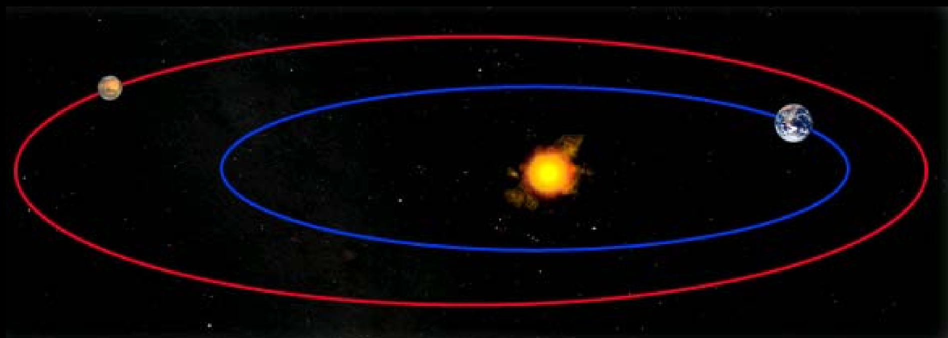
Known

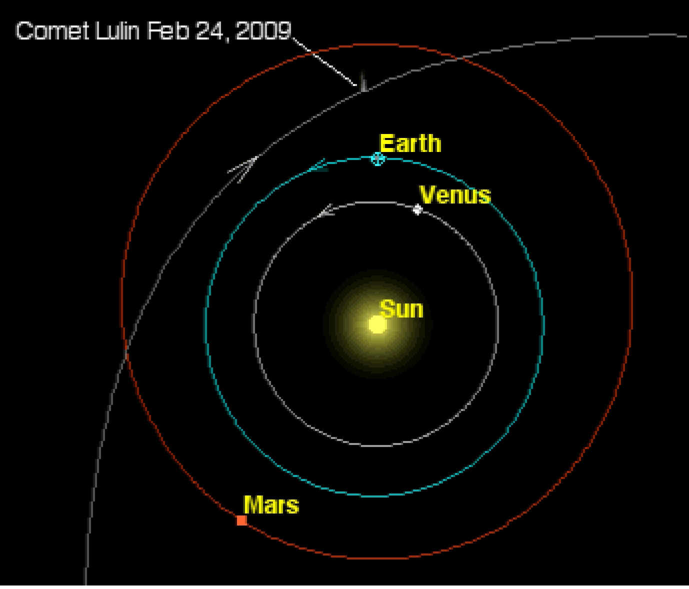
Wonderful Math Tools


y axis
ORBIT OF MARS
SATURN
JUPITER
URANUS
NEPTUNE
PLUTO
x axis


Comet Halley
~76 years
Comet Tempel 1
~5.5 years
Comet Hyakutake
~17,000 years
Mercury
Venus
Earth
Mars
Jupiter
Saturn
Uranus
Neptune
Pluto

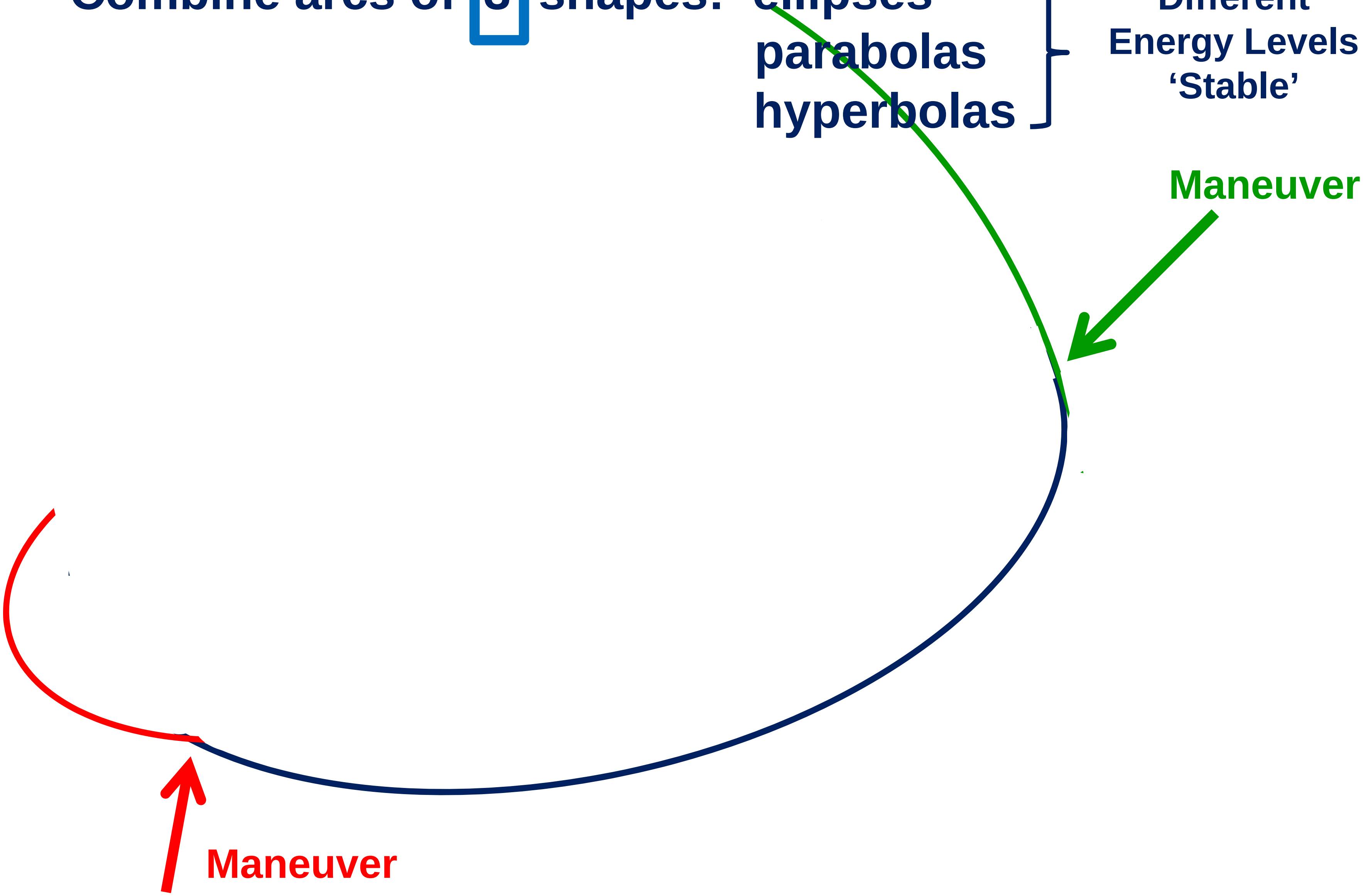

Jupiter
comet

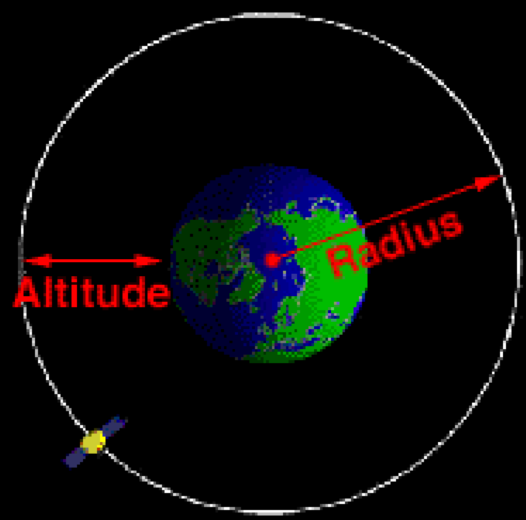


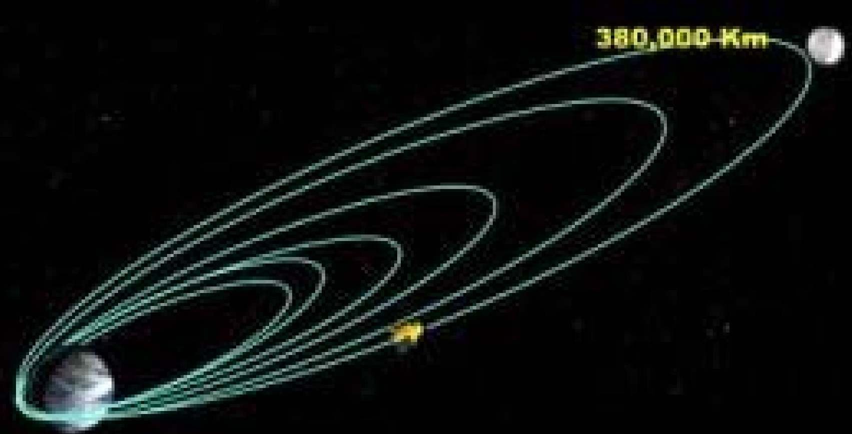

Comet Lulin Feb 24, 2009
Earth
Venus
Sun
Mars

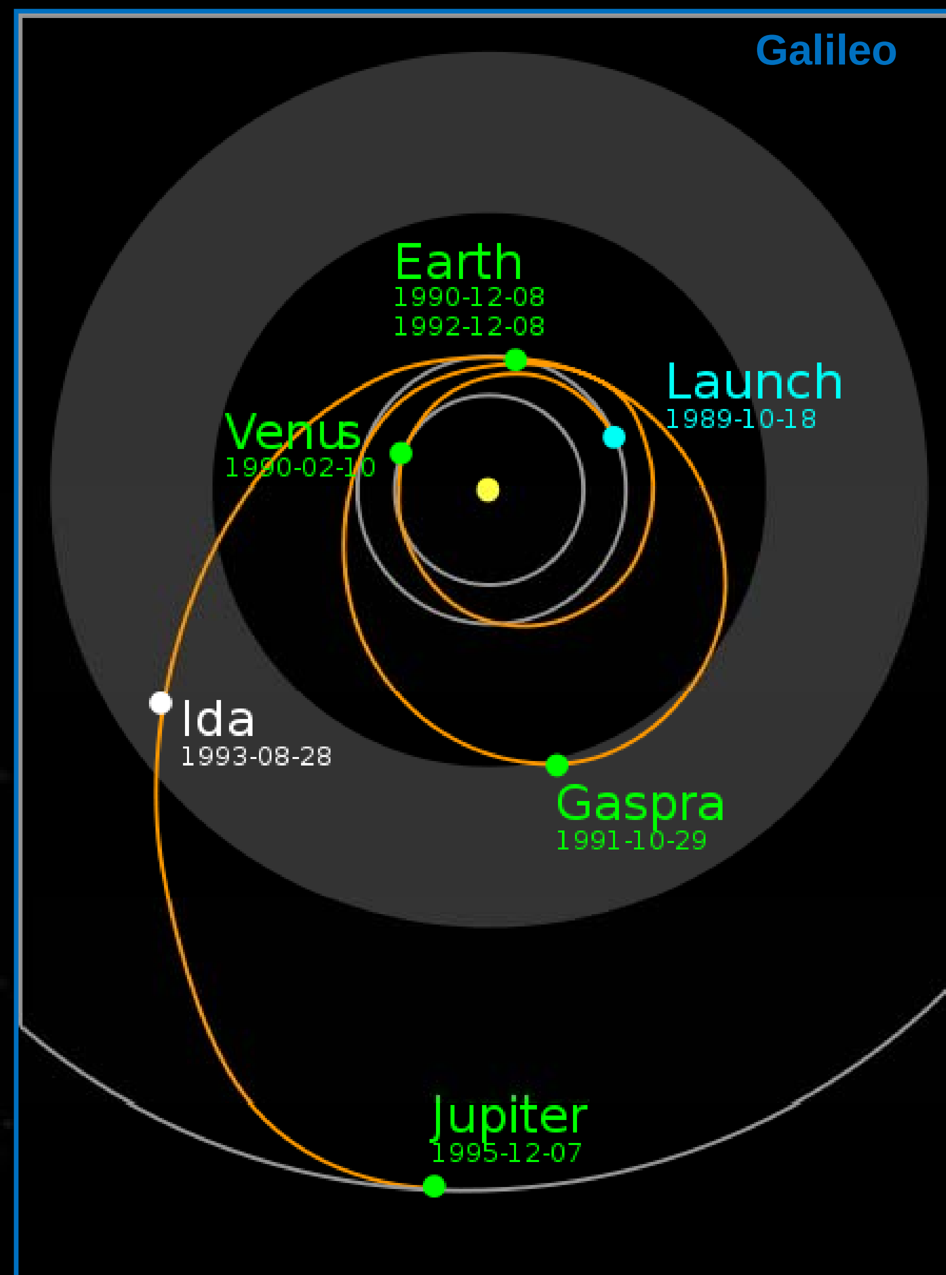
Trajectory Design

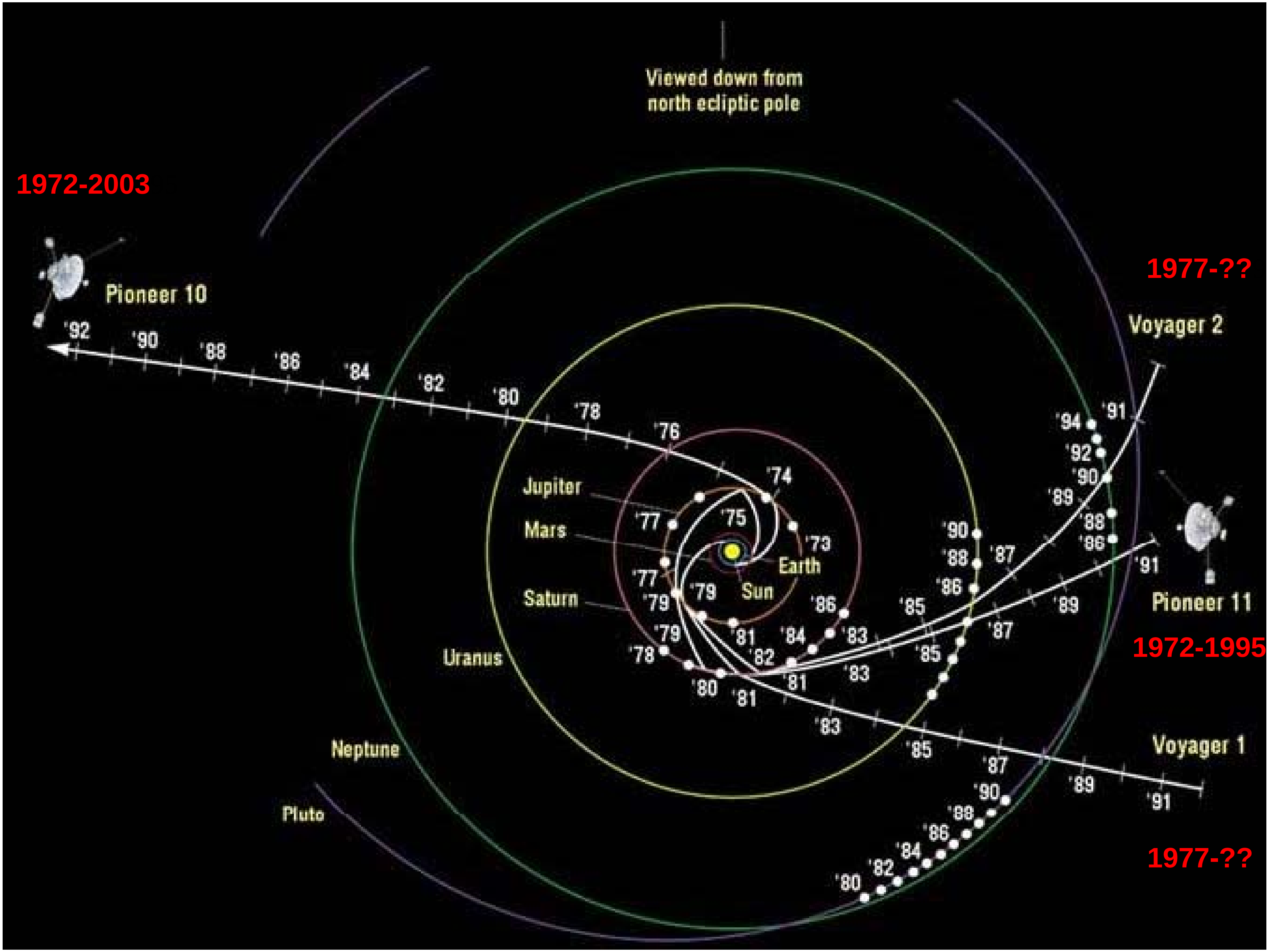
Combine arcs of $\boxed{3}$ shapes: ellipses, parabolas, hyperbolas — Different Energy Levels ‘Stable’

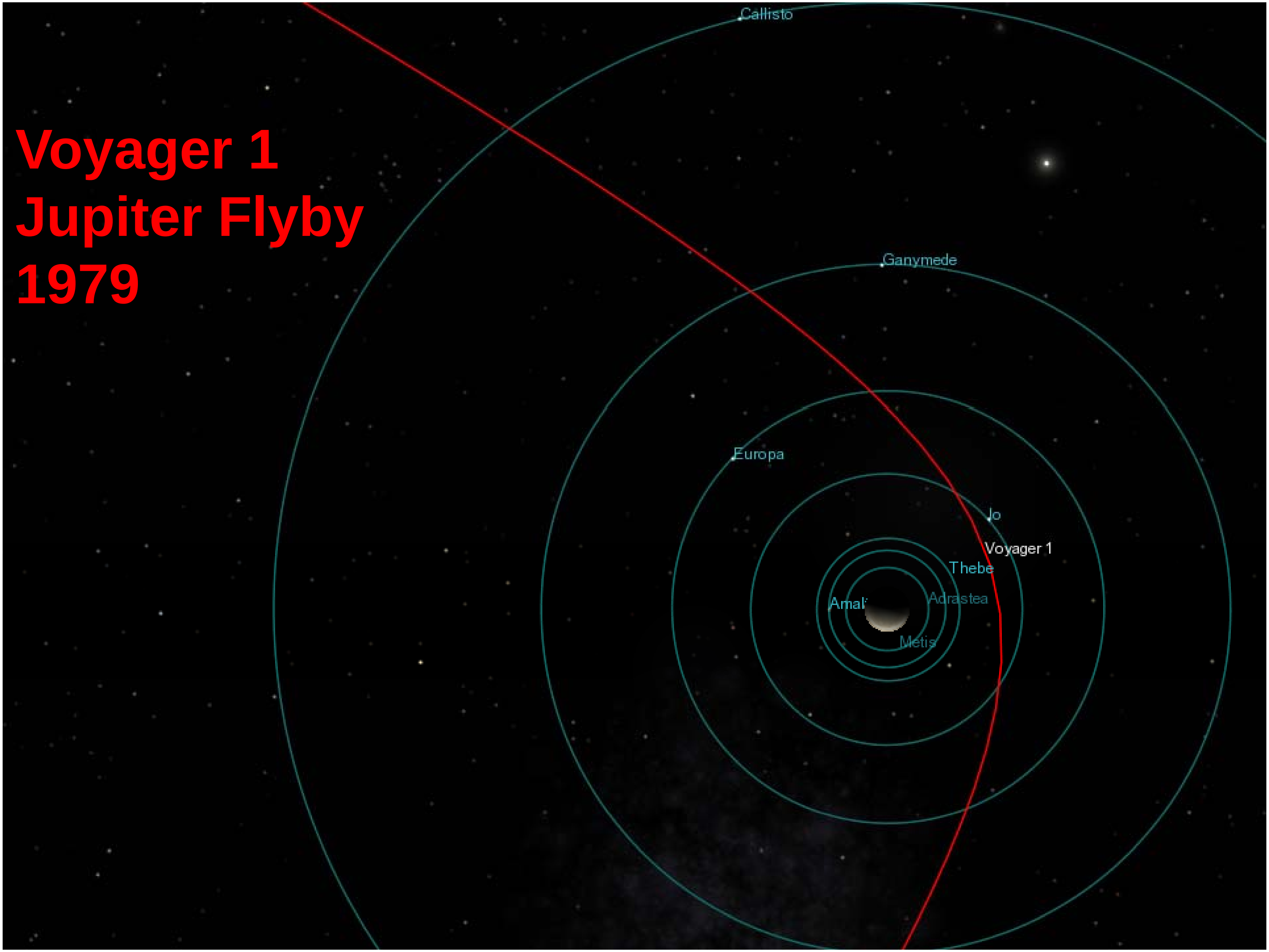



Altitude
Radius

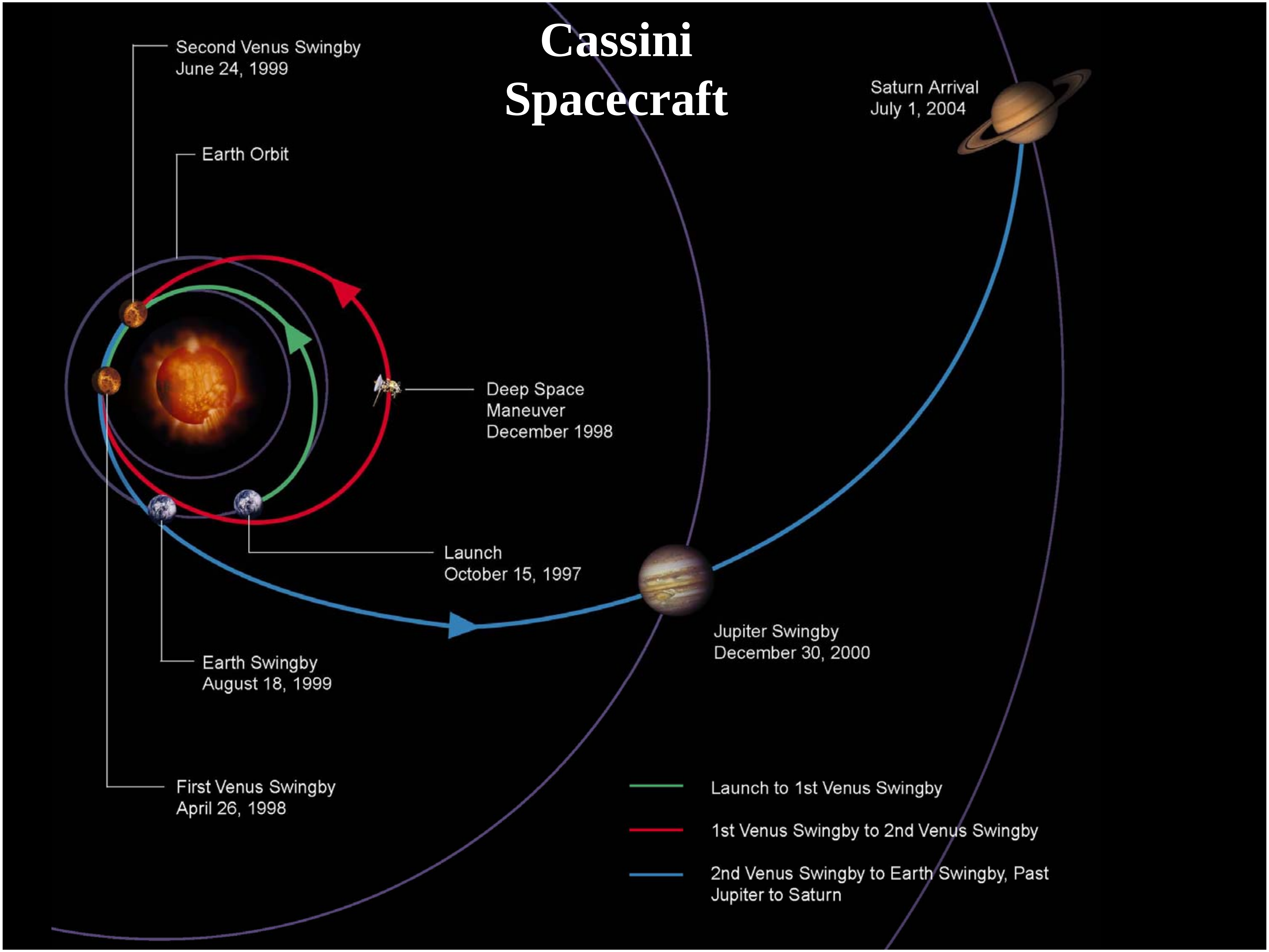

380,000 Km

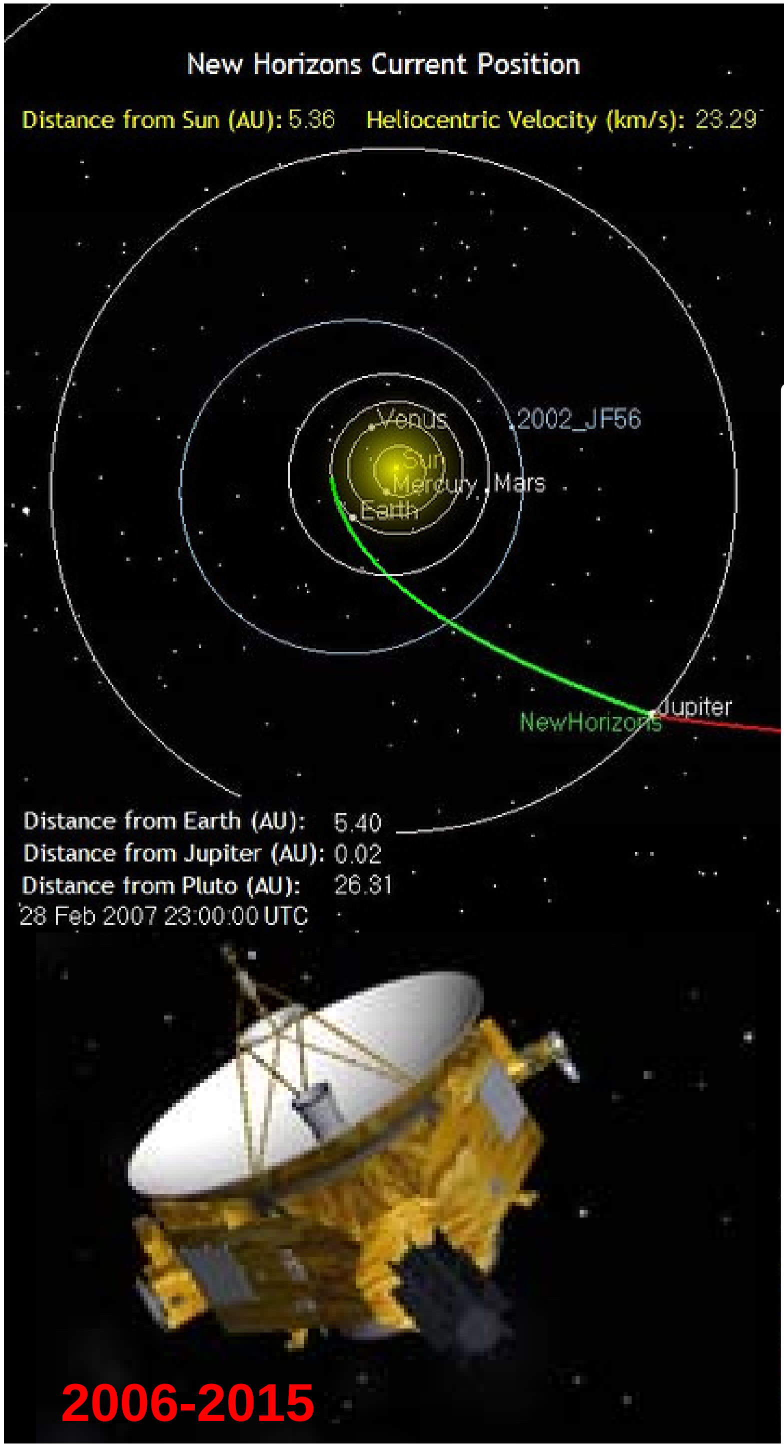

Galileo
Earth
1990-12-08
1992-12-08
Launch
1989-10-18
Venus
1990-02-10
Ida
1993-08-28
Gaspra
1991-10-29
Jupiter
1995-12-07


Viewed down from
north ecliptic pole
1972-2003
Pioneer 10
'92
'90
'88
'86
'84
'82
'80
'78
'76
'74
'75
'73
'77
'79
Jupiter
Mars
Earth
Sun
Saturn
Uranus
Neptune
Pluto
1977-??
Voyager 2
'94
'91
'92
'90
'89
'88
'86
'87
'85
'83
'84
'81
'82
'80
'78
Pioneer 11
1972-1995
Voyager 1
'89
'91
1977-??


Voyager 1
Jupiter Flyby
1979
Callisto
Ganymede
Europa
Io
Voyager 1
Thebe
Adrastea
Amal
Metis

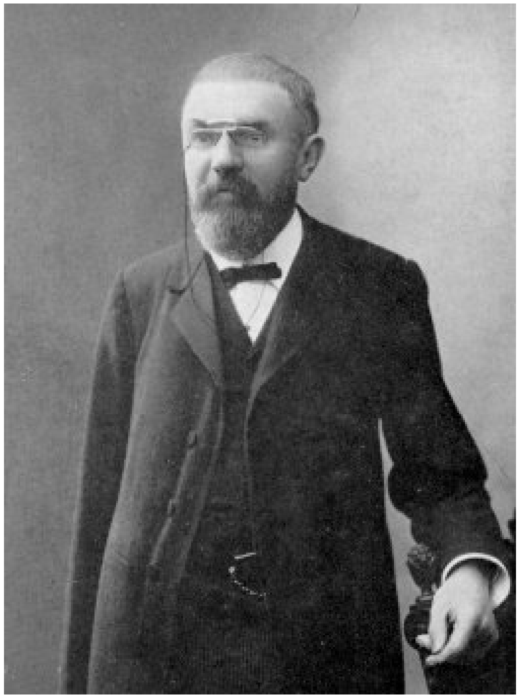
Cassini Spacecraft



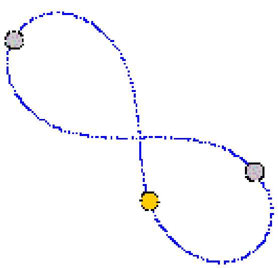


Yet, demands for space vehicles increasingly complex

→ our understanding of motion in the solar system is actually incomplete



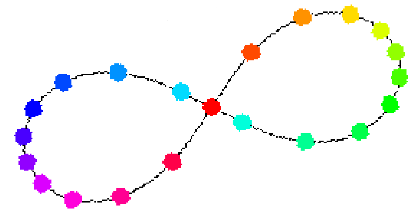
Poincaré (1854-1912)



Dynamical Chaos

Poincaré first glimpsed $\boxed{\text{chaos}}$ in the gravitational problem in mid-1880's

Contest: Solve for motion of N Bodies



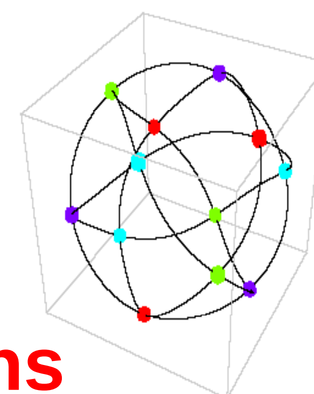
Poincaré did not solve (not even N = 3)

Prize for understanding + many new ideas

Three-volume memoir

- **foundation for several branches of math**
- **new approach**

New era in celestial mechanics



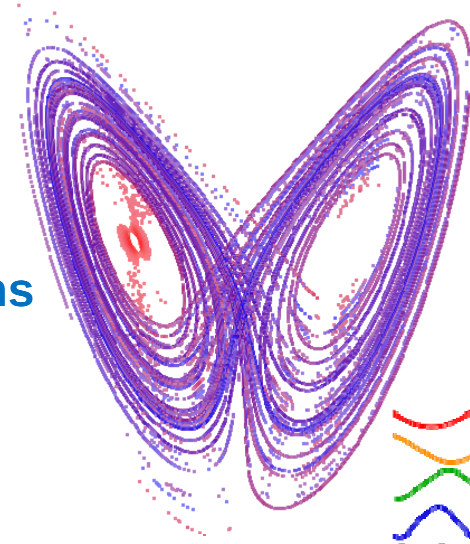
Poincaré as visionary:
sensitive dependence on initial conditions

Deterministic Chaos

Contradiction in terms?
Wild, unpredictable behavior?

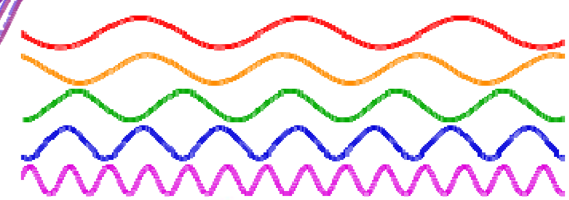
NO!

Dynamical systems theory and *chaos*
→ long-term behavior
typically quite complicated



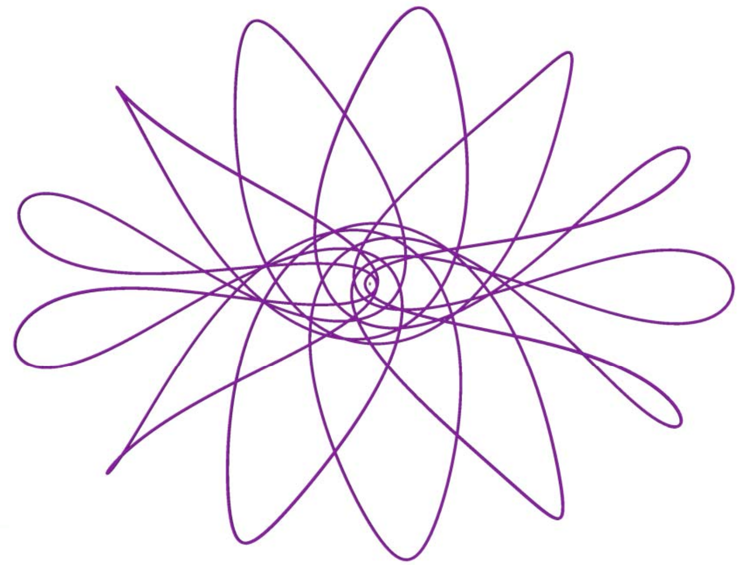
Properties:

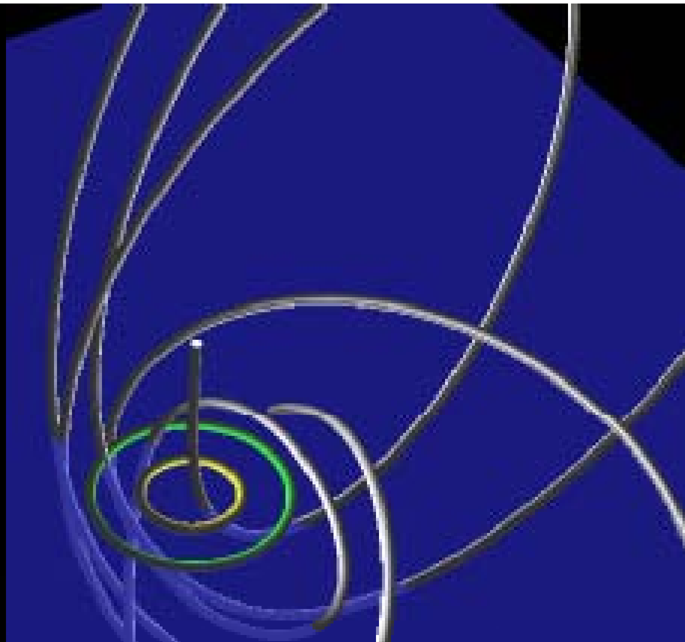
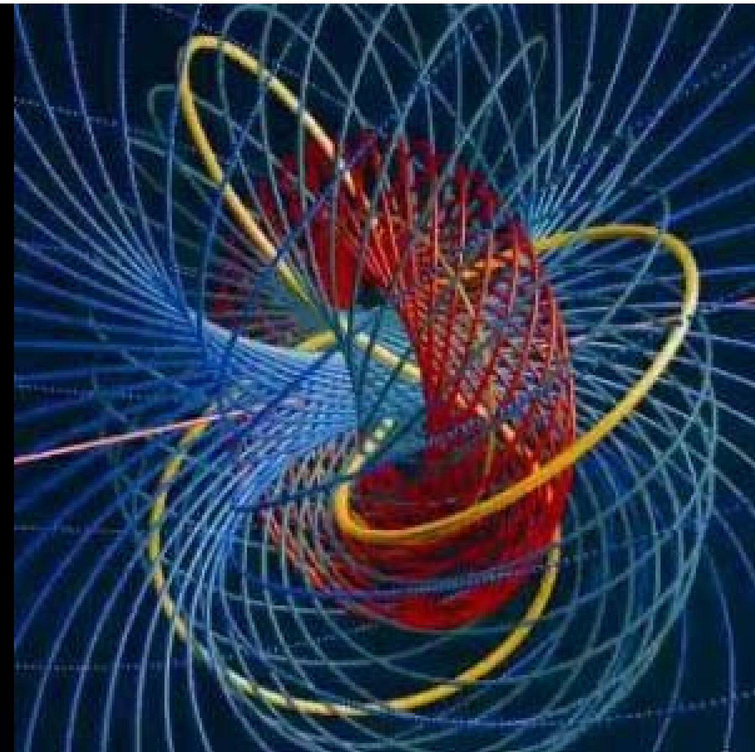
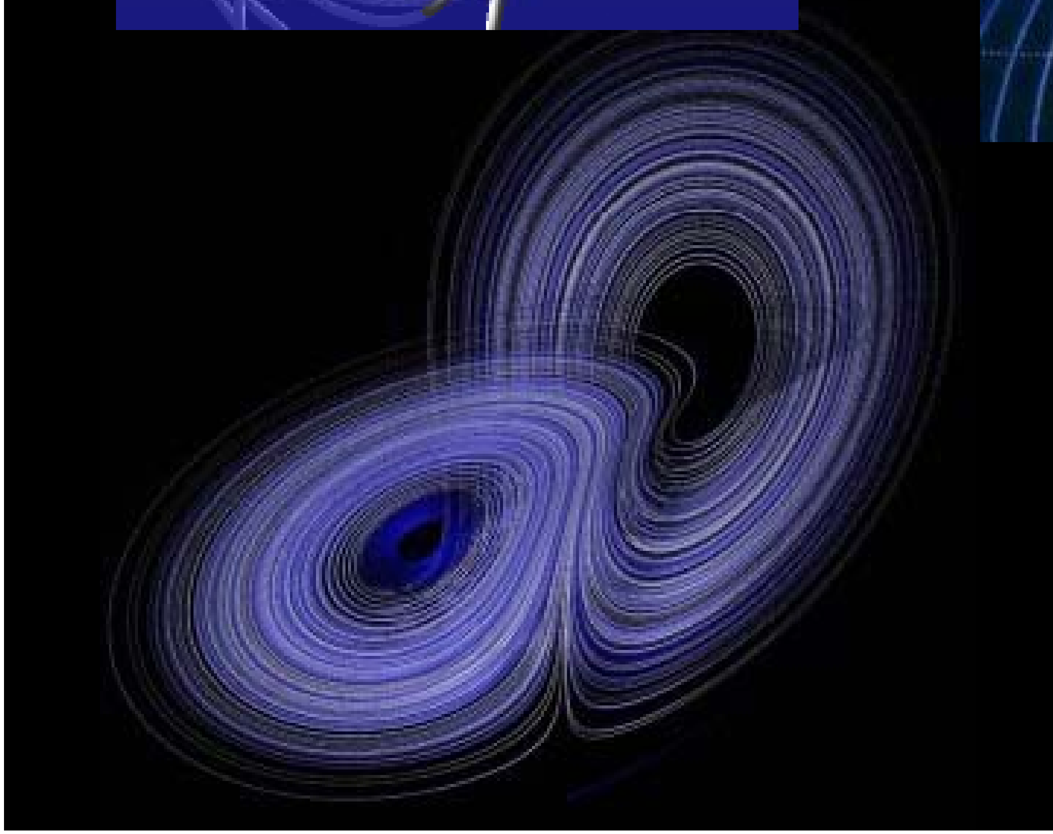
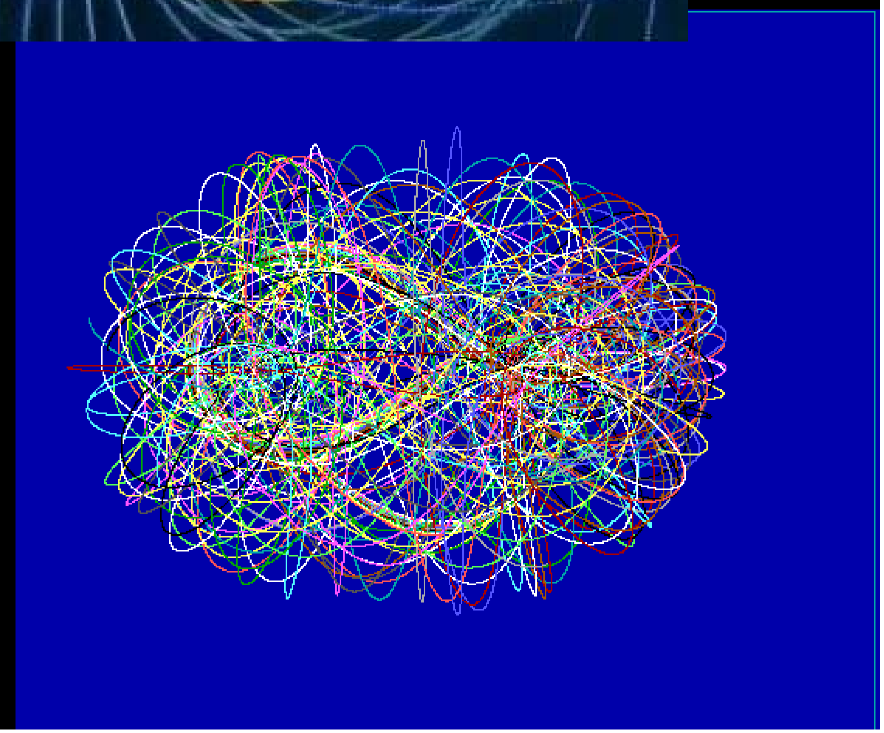
1. Sensitive to initial conditions
   Minor changes cause huge fluctuations
2. Many frequencies are excited
3. Periodic orbits must be dense
   System appears unpredictable
4. Behavior must be <u>locally</u> <u>unstable</u>;
   global stability



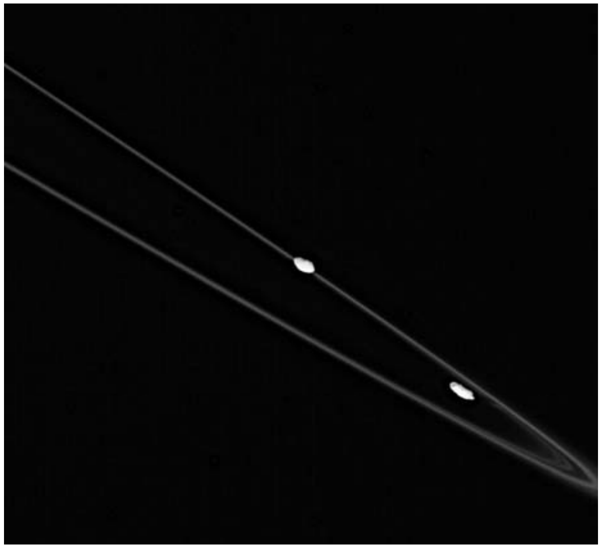
Goal: *fixed points*
periodic points

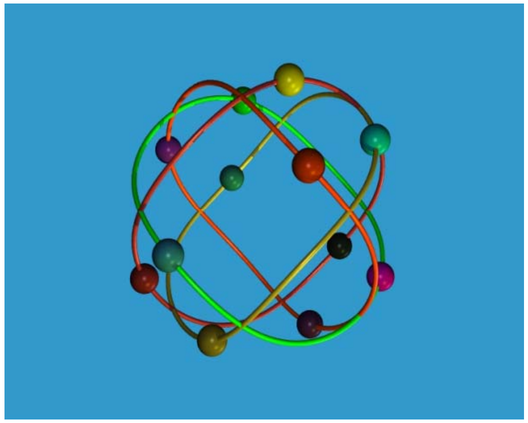
Both can be attractive



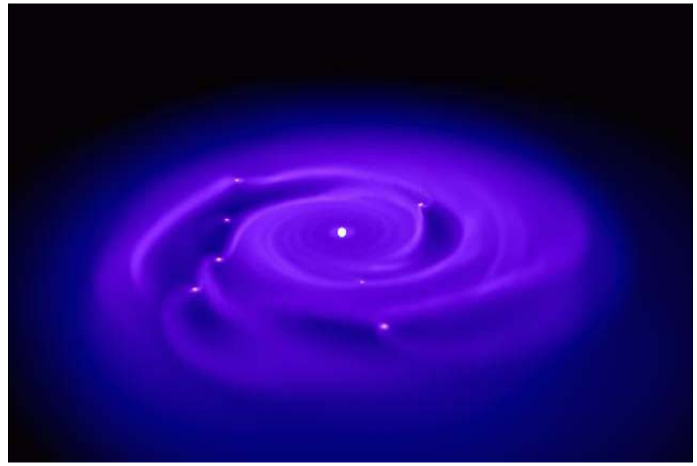






Earth-Moon Distance: 384,000 km
Earth Scale: 3x
Moon Scale: 5x
Broader Understanding
Astrodynamics Design Opportunity!!
Infinite number of types of arcs!





Mathematics



Celestial Mechanics

Stellar Dynamics

N-Body Problem

Mathematicians interested forever

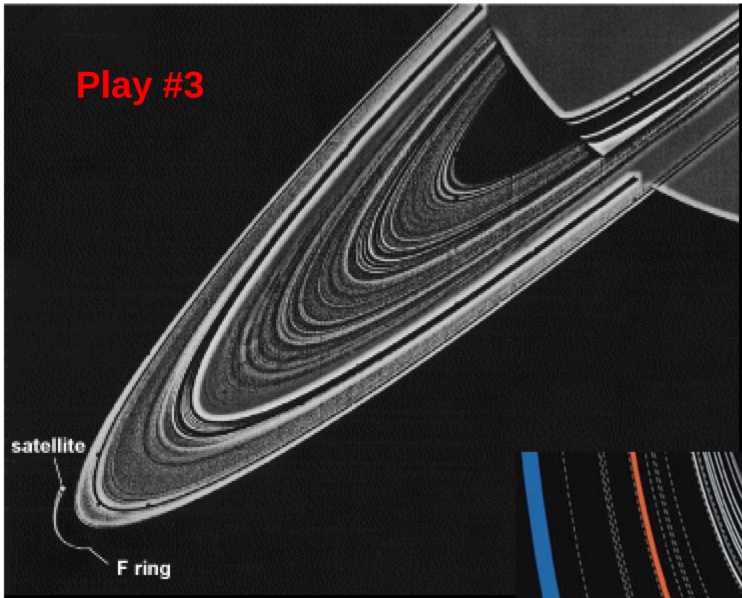
Renewed interest: celestial mechanics and stellar dynamics

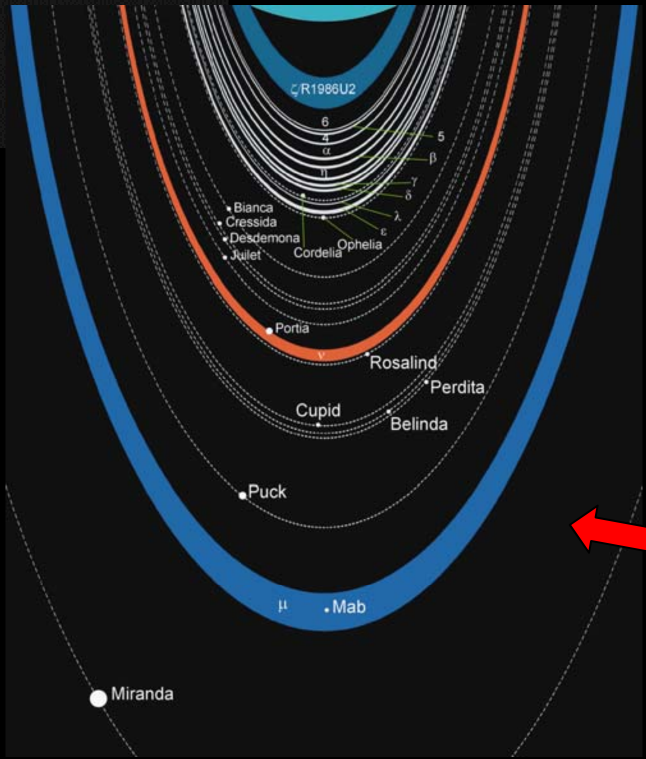
Few bodies ⟺ Computers ⟺ Statistical

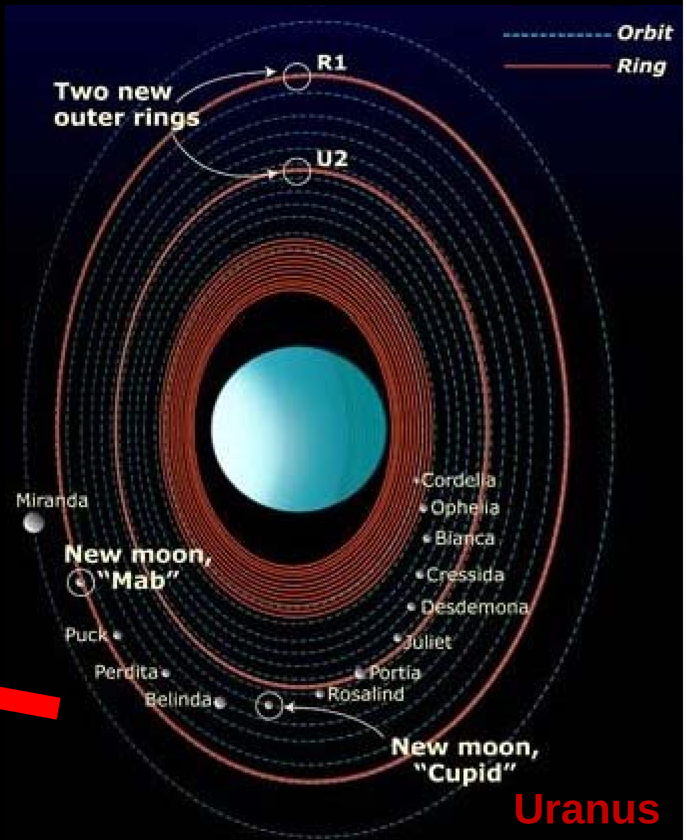
Compute each orbit

Examples of Natural Motion modeled in terms of multiple bodies

Play #3





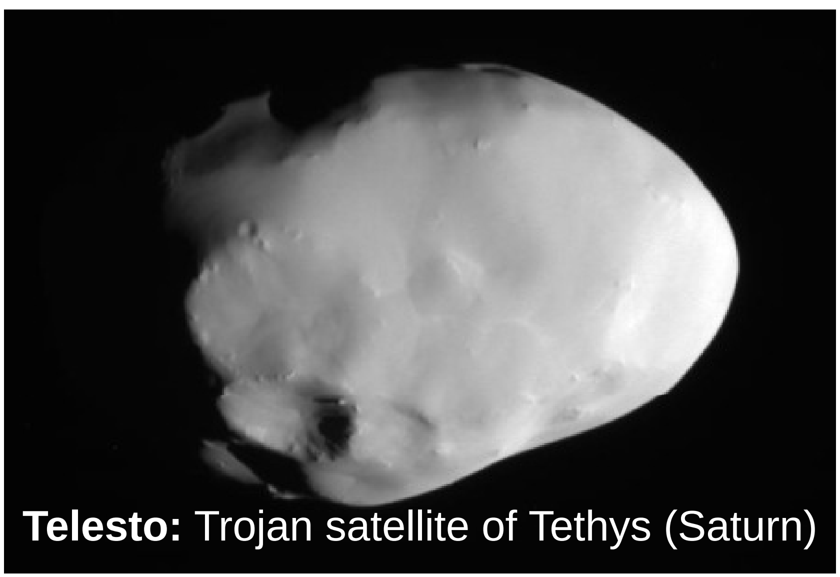


Uranus

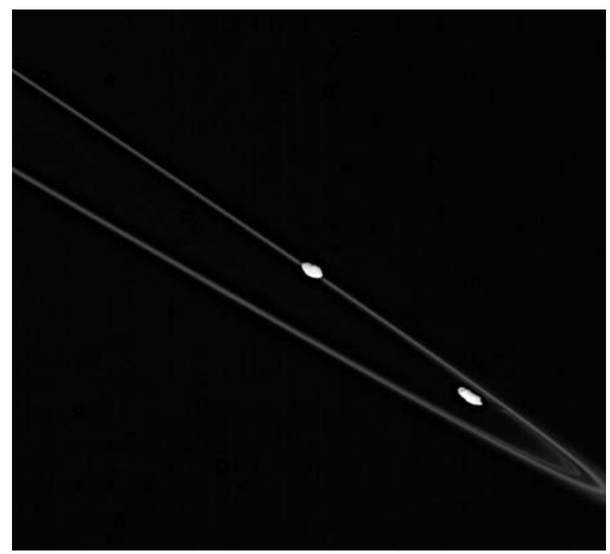
Astrodynamics: N-Body Problem

Where are we?

- Simplest system can have both regular and chaotic behavior
- Laplace Universe – gigantic and perfect watch – has disappeared
- Poincaré – Dynamical Systems + ‘chaos’
- Opened new opportunities
  - **Examples of natural motion modeled in terms of multiple bodies**



Telesto: Trojan satellite of Tethys (Saturn)



Shepherd Moons - Saturn

Astrodynamics: N-Body Problem

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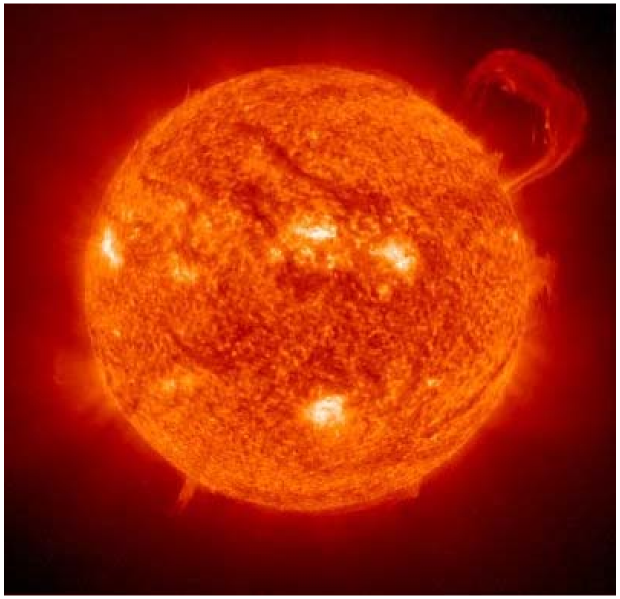
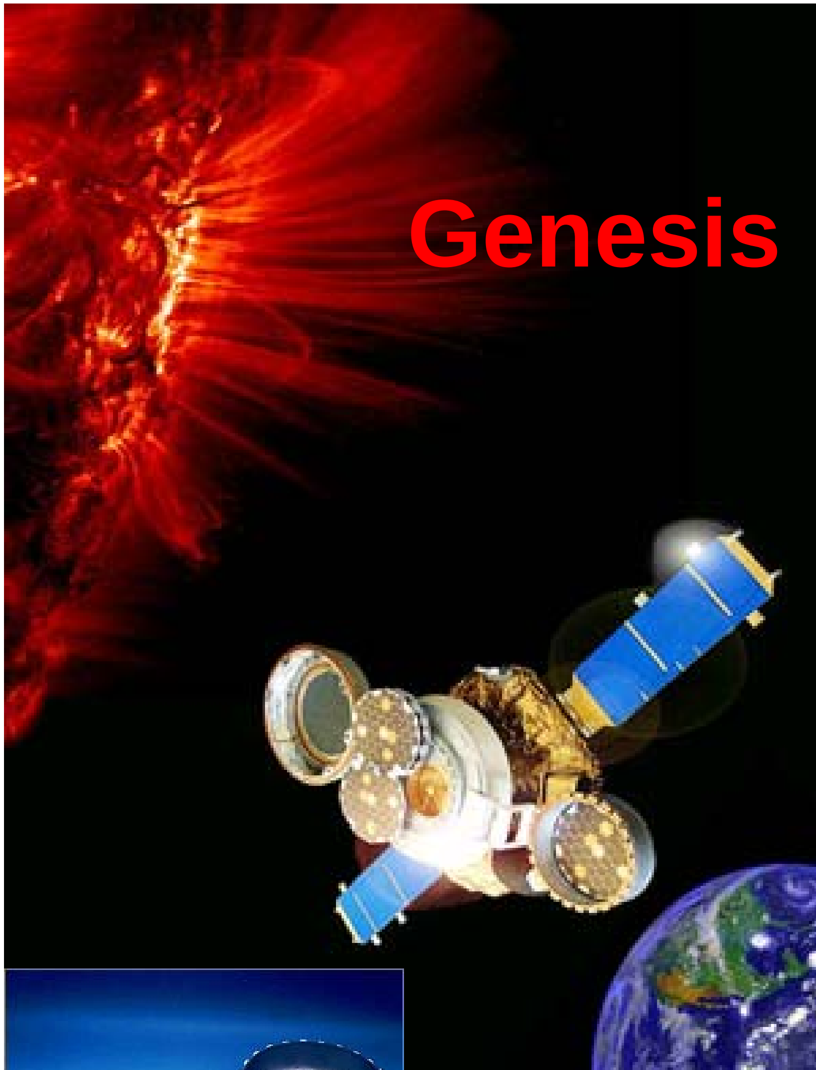
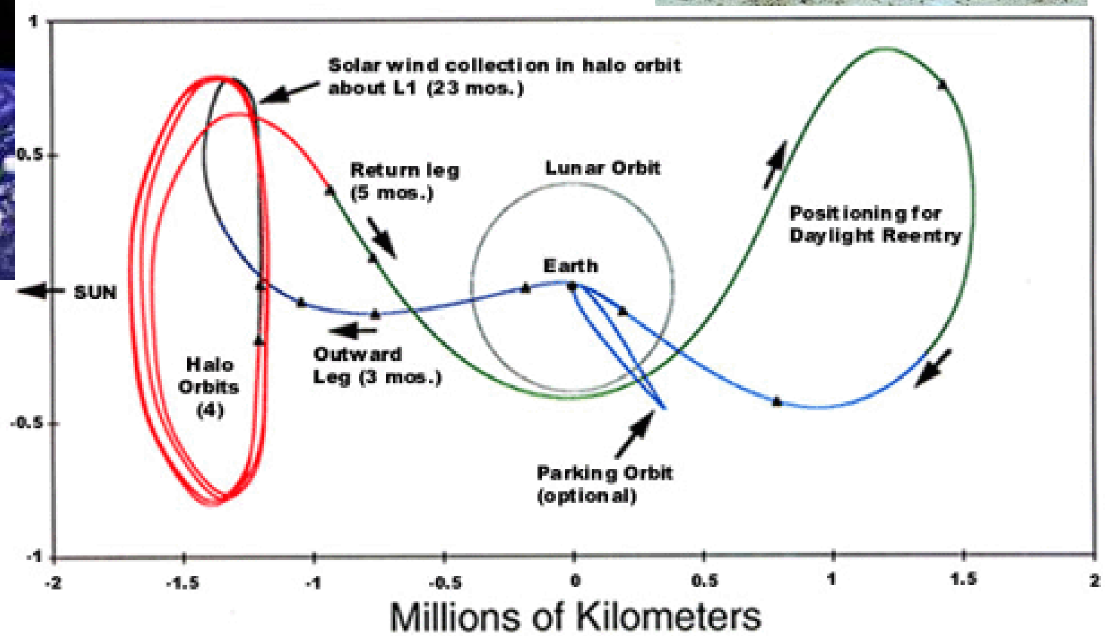


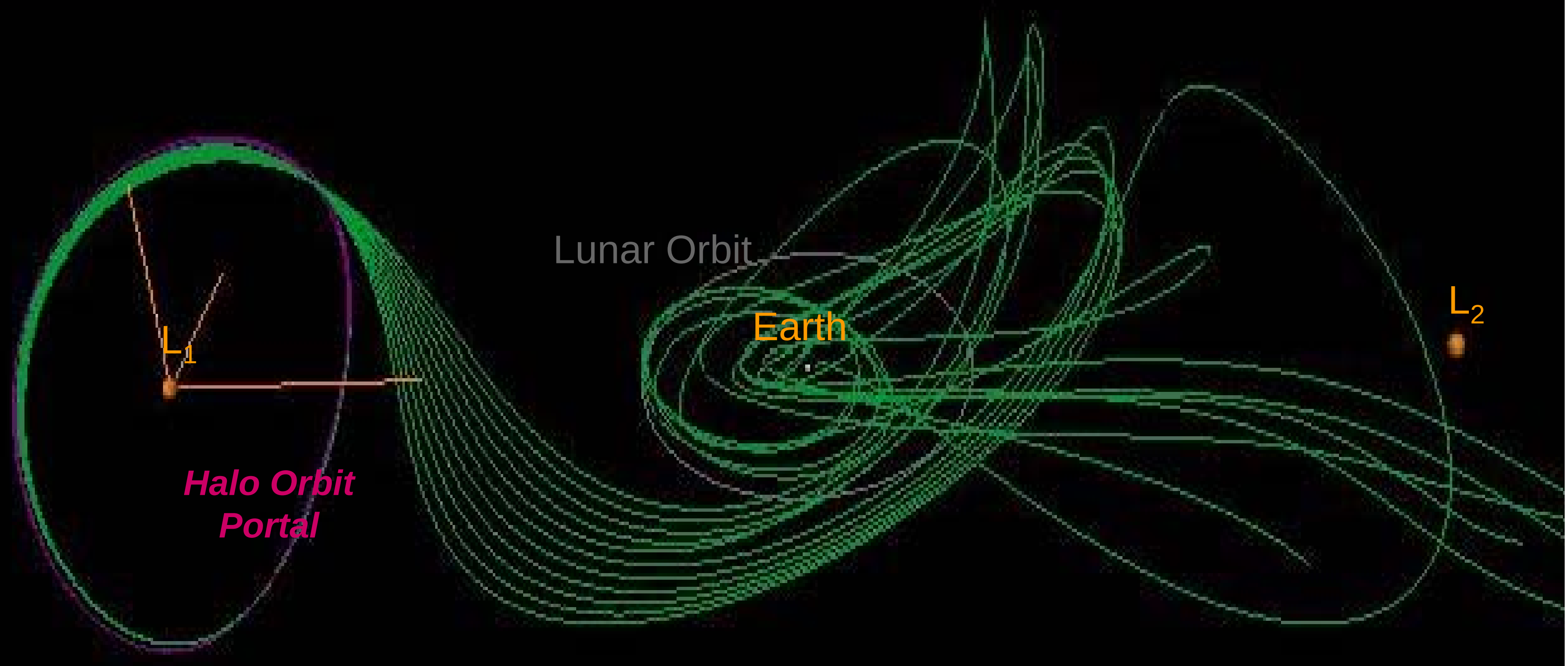
Phenomena that affects Earth


Genesis

2001-2004


Solar wind collection in halo orbit about L1 (23 mos.)
Return leg (5 mos.)
Lunar Orbit
Positioning for Daylight Reentry
Earth
SUN
Outward Leg (3 mos.)
Halo Orbits (4)
Parking Orbit (optional)
1
0.5
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Millions of Kilometers

Genesis Trajectory Design



Astrodynamics: N-Body Problem

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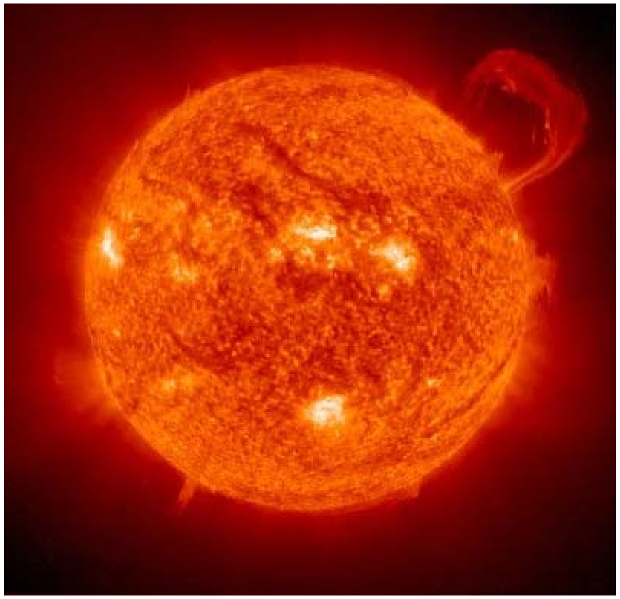
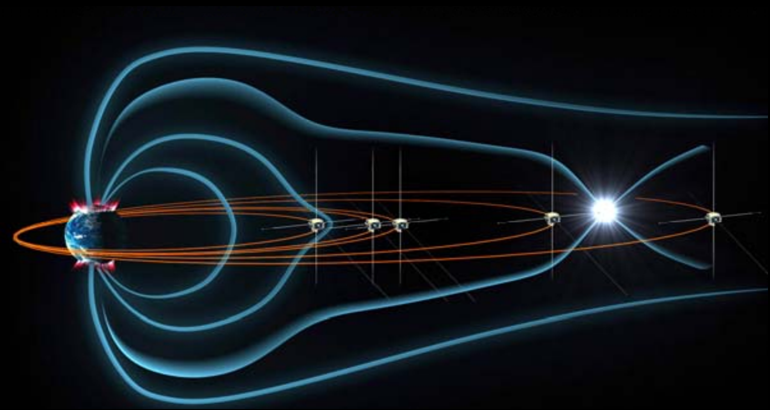
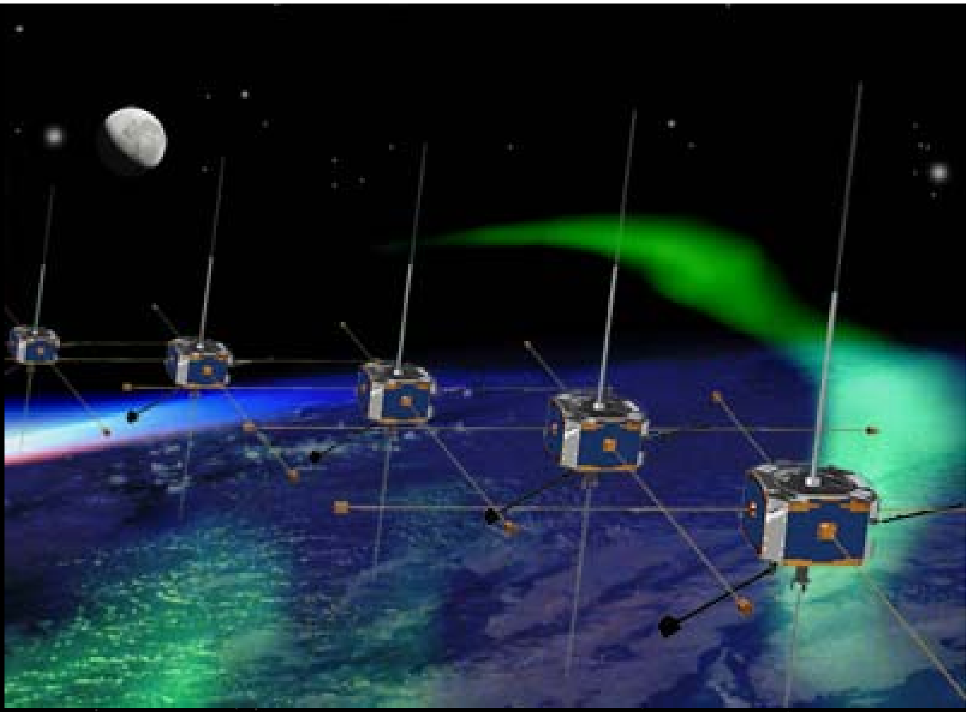


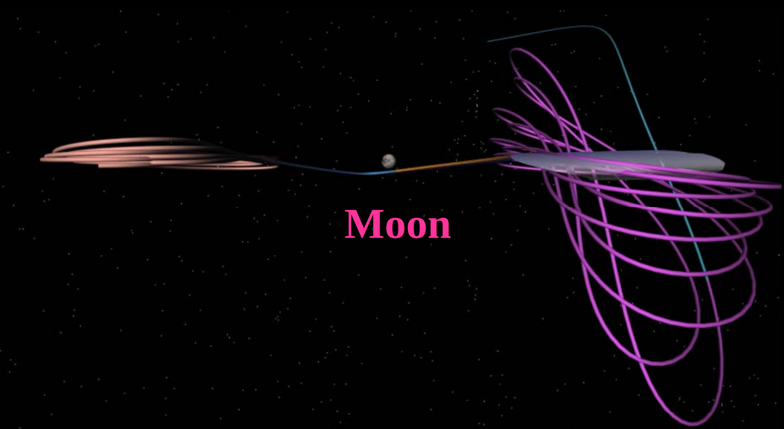
Phenomena that affects Earth

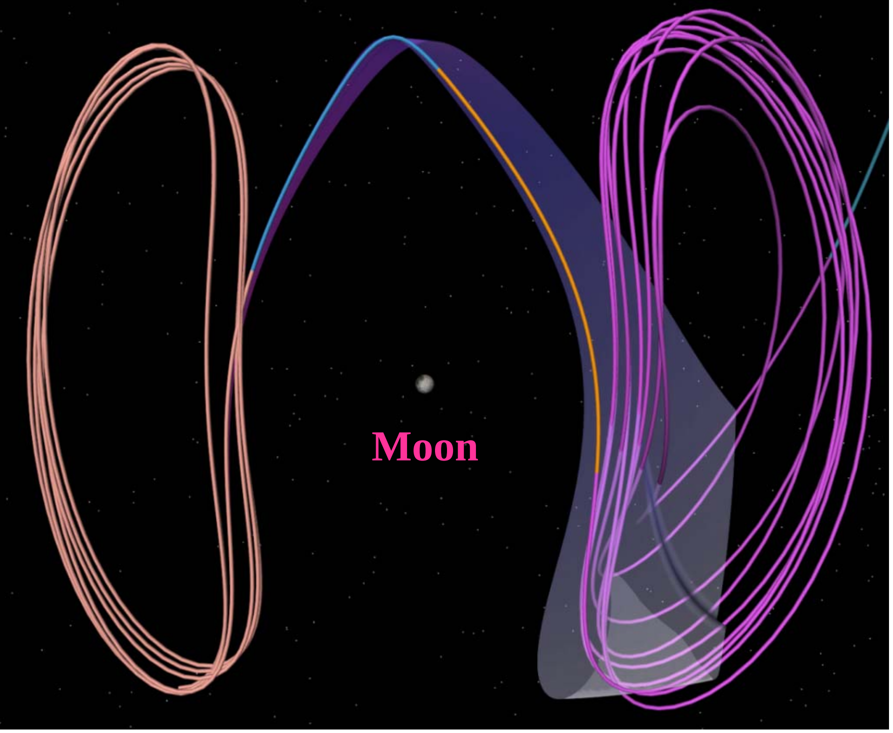
Artemis

Physics of Northern Lights to Lunar Wake

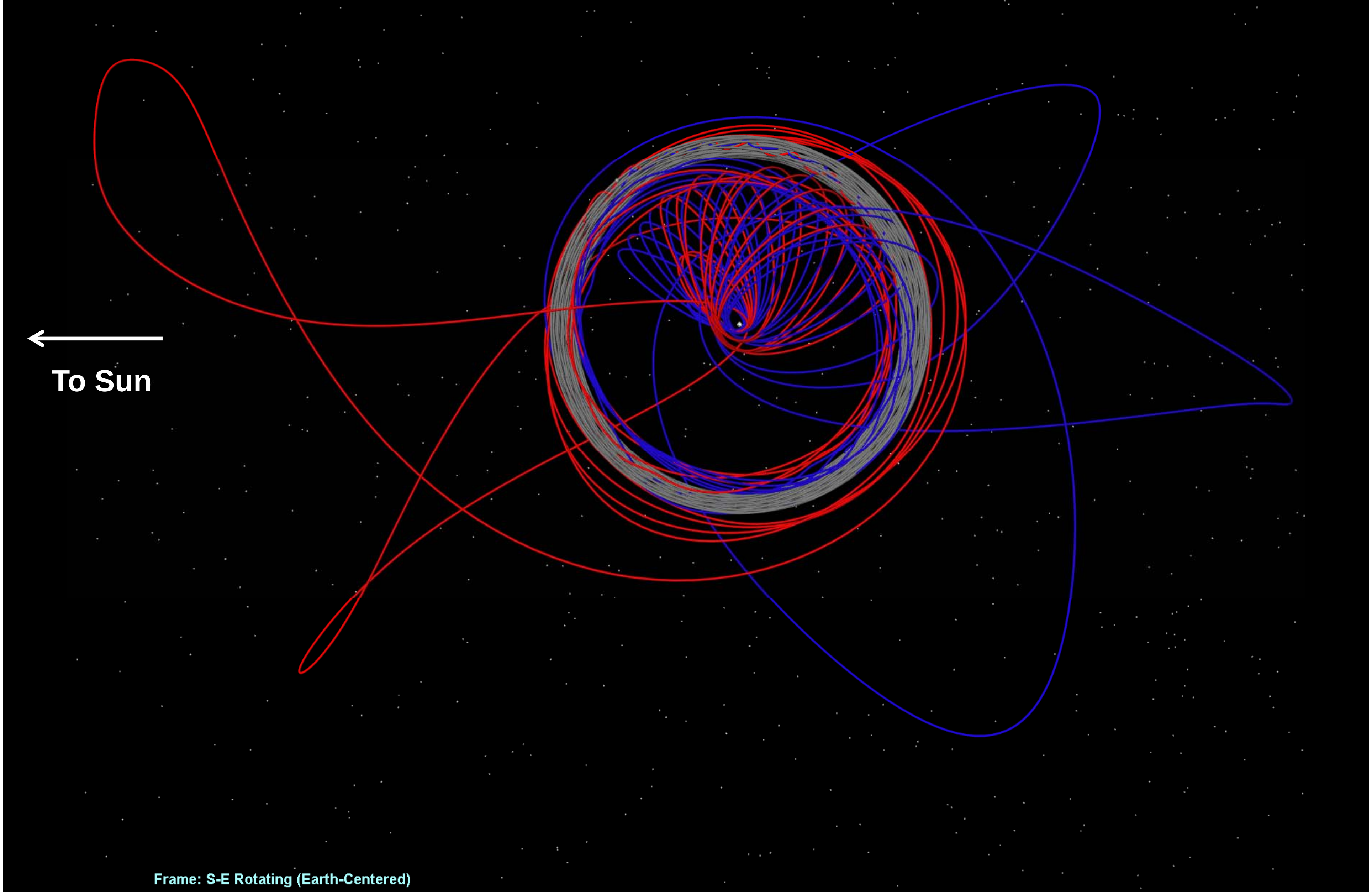








Artemis P1 /P2 Baseline Trajectory



Astrodynamics: N-Body Problem

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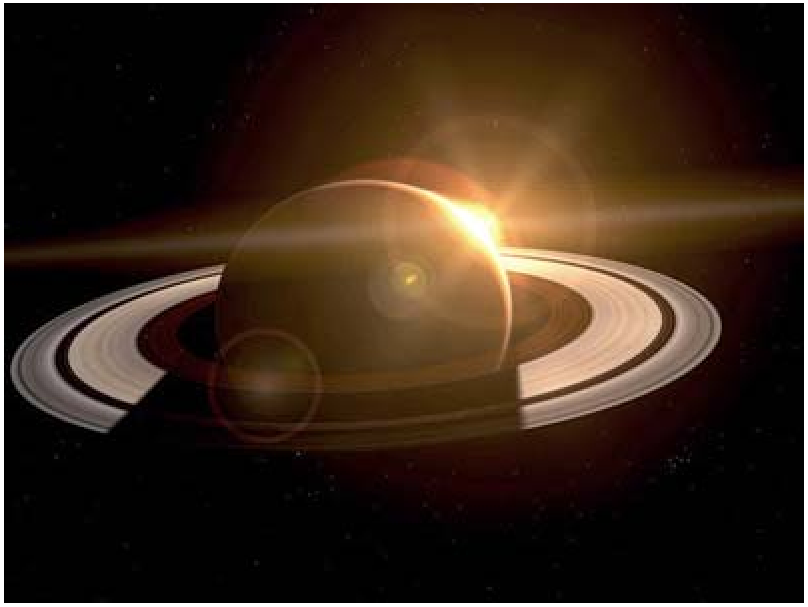
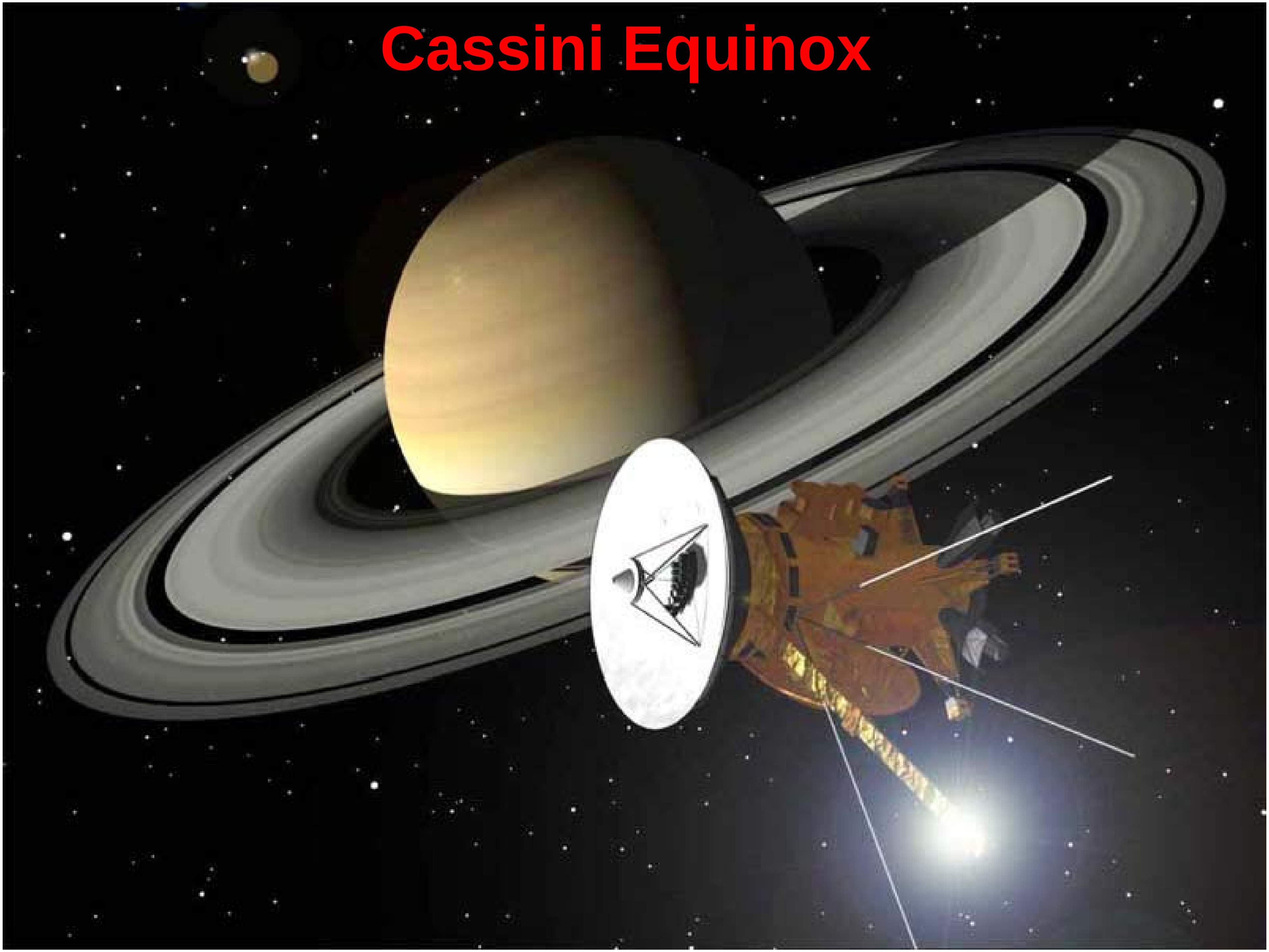
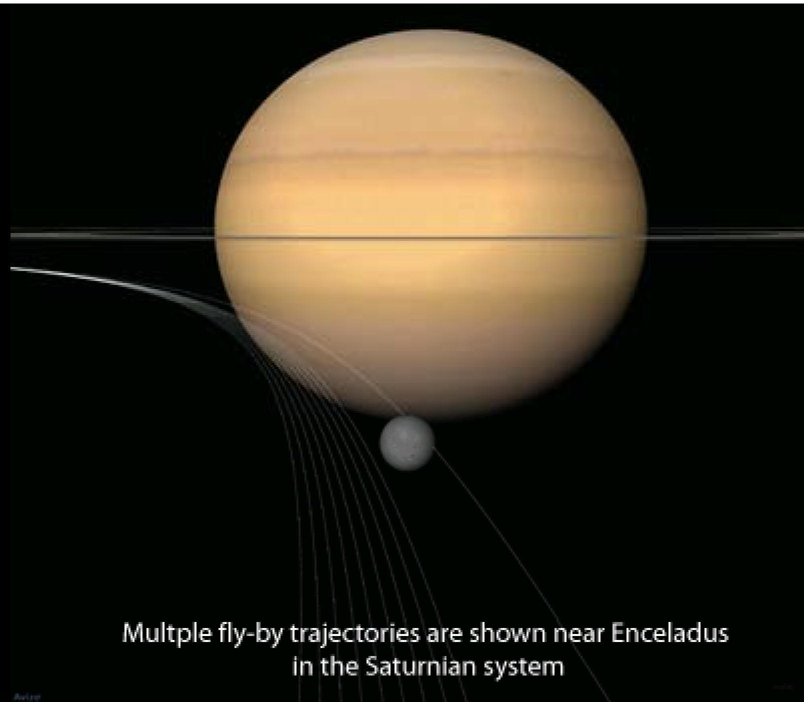
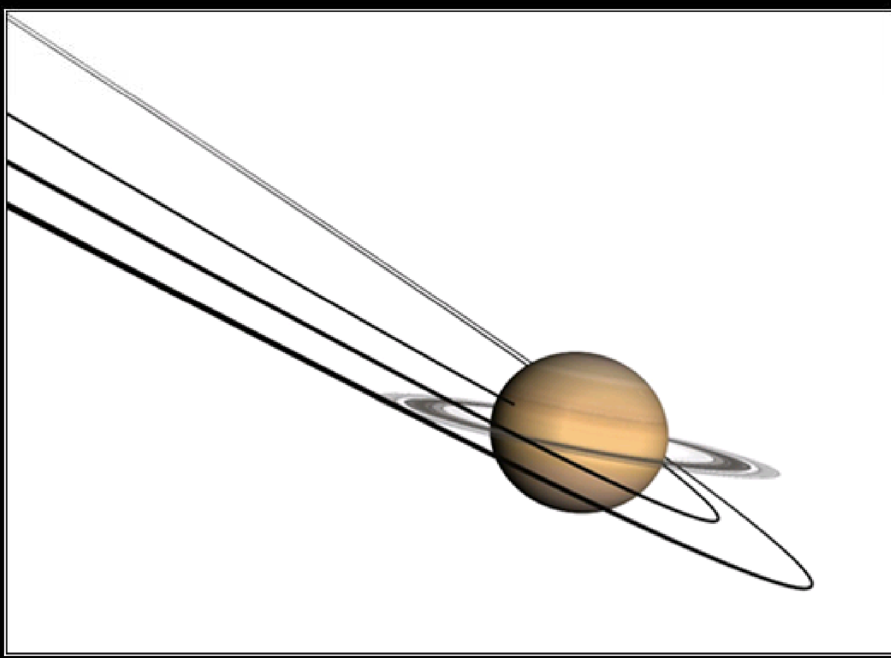


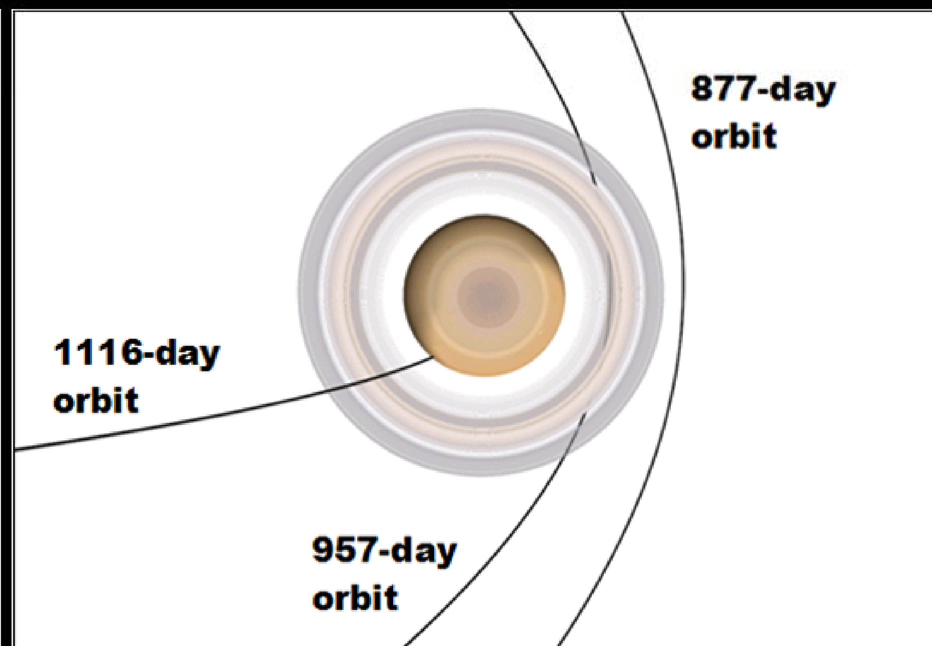
Knowledge of our Earth
+ solar system

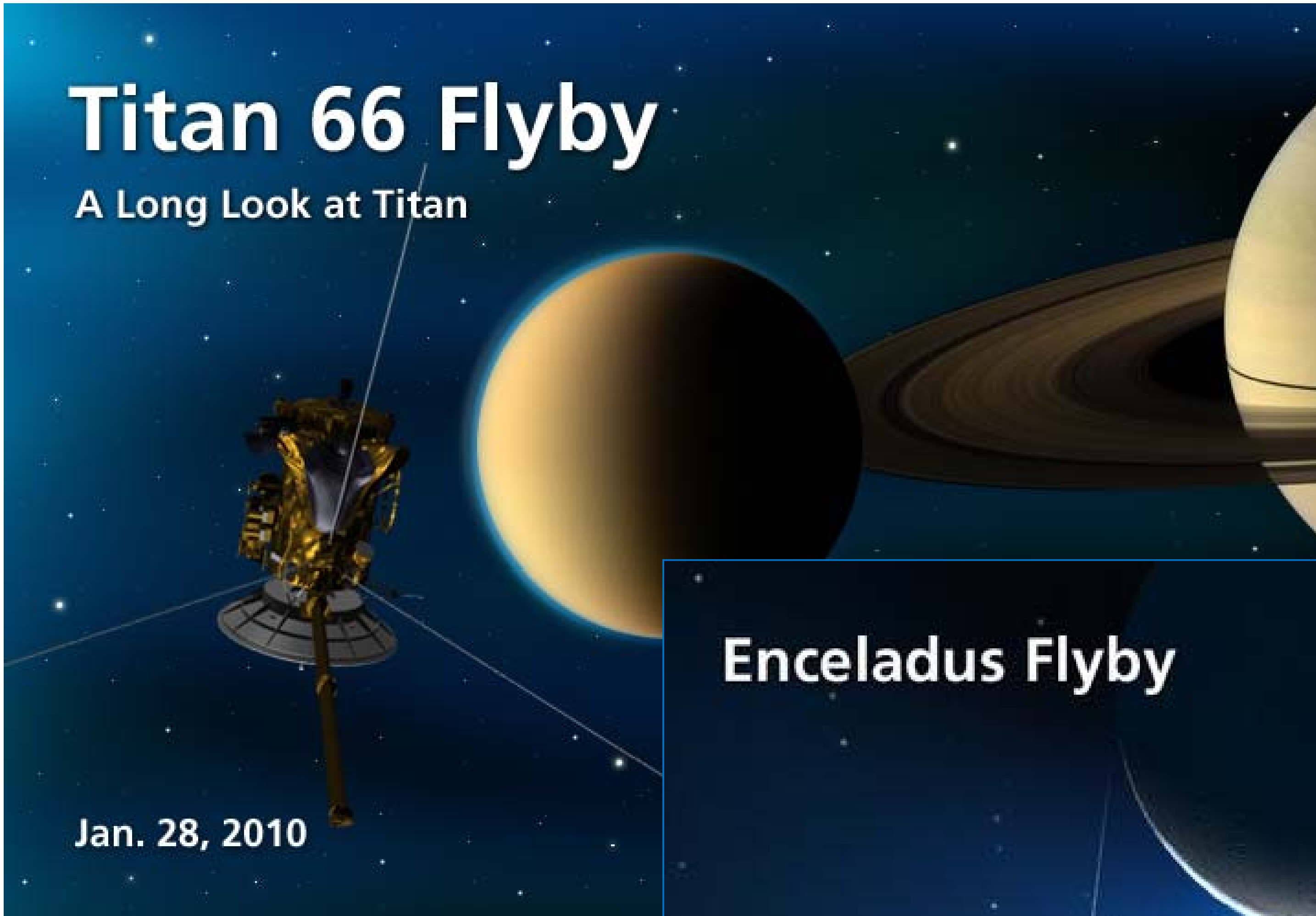

Cassini Equinox



Multple fly-by trajectories are shown near Enceladus in the Saturnian system





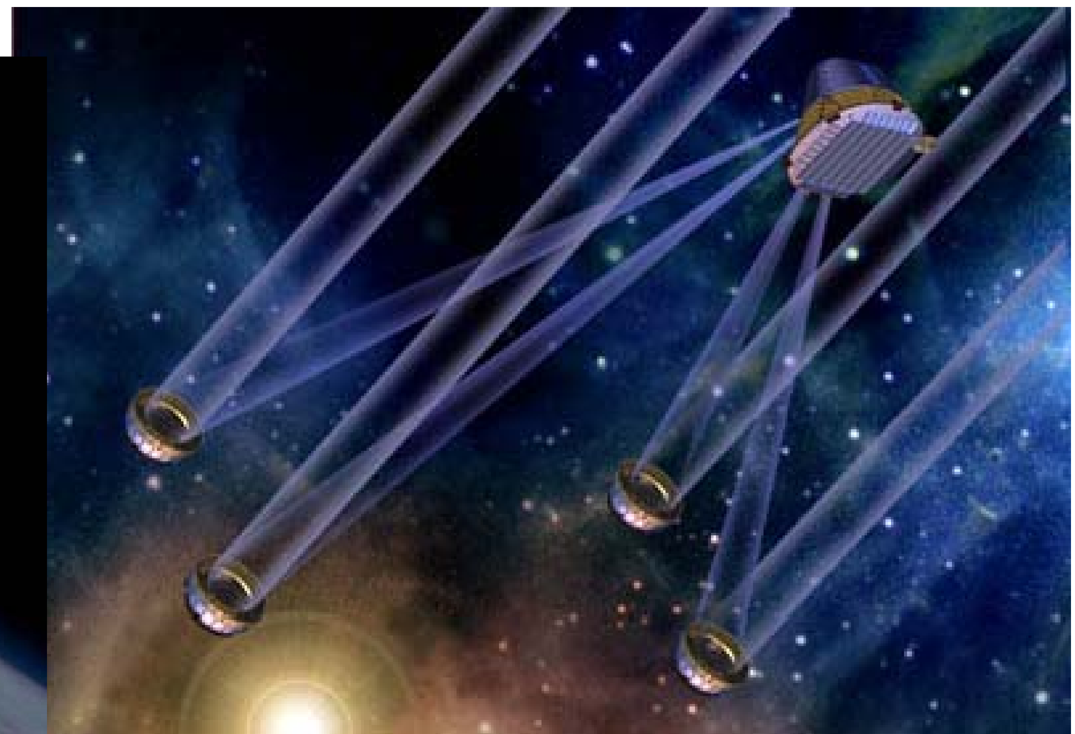

Titan 66 Flyby
A Long Look at Titan
Jan. 28, 2010


Enceladus Flyby
The Plume's the Thing
May 18, 2010

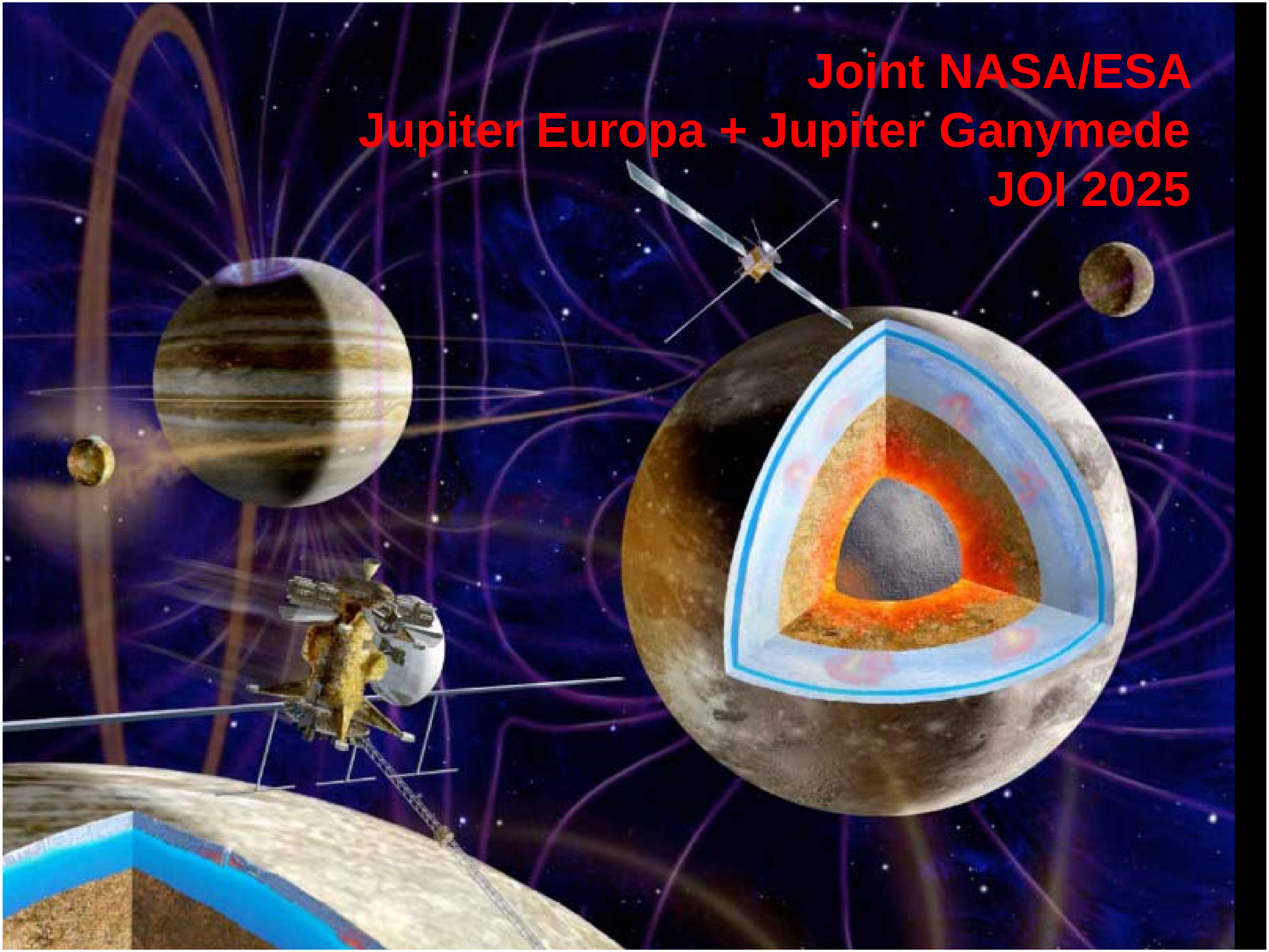
Astrodynamics: N-Body Problem

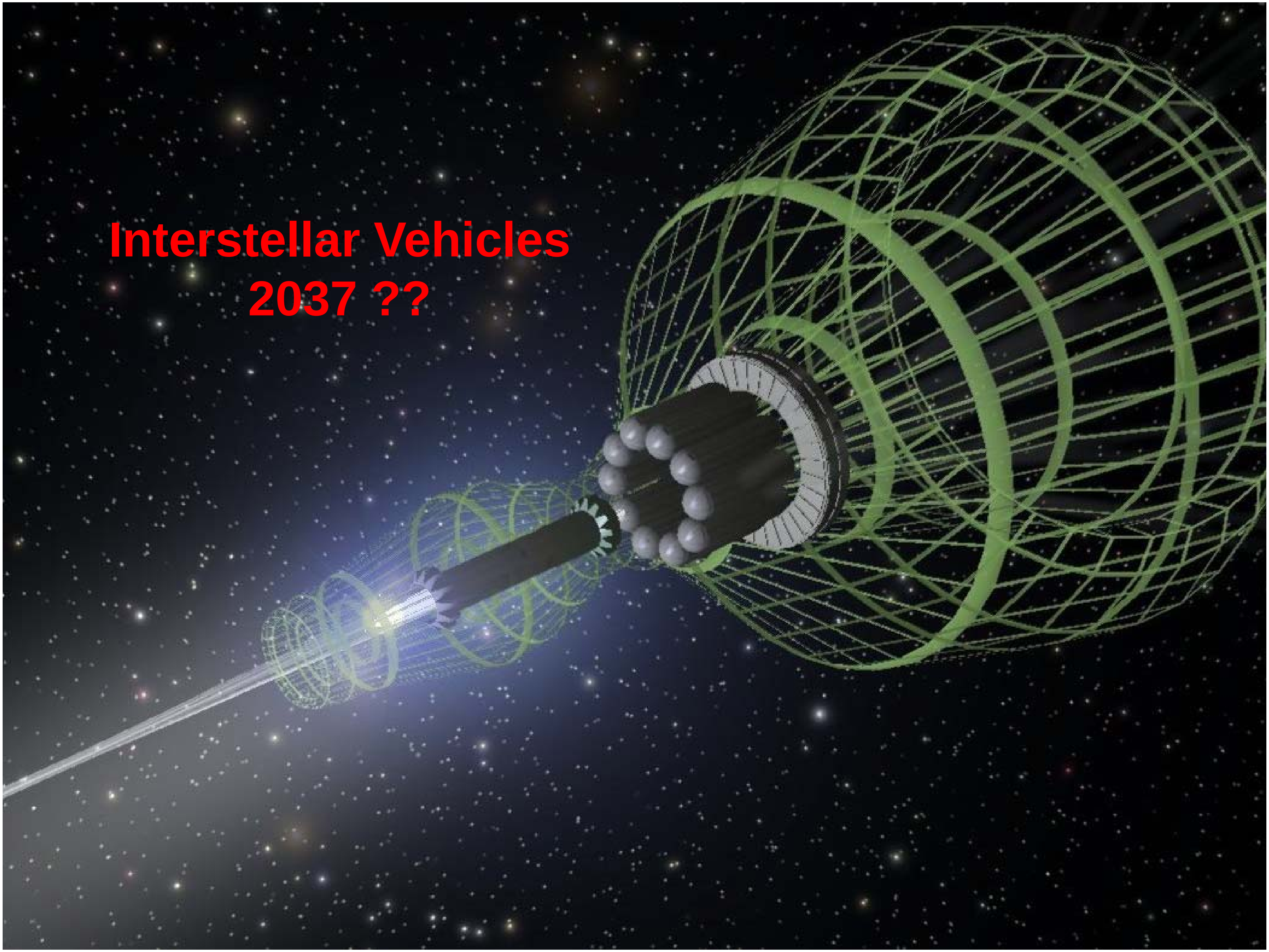
- **Poincaré:** "real aim of celestial mechanics is not to calculate the ephemerides but to recognize if Newton's law is sufficient to explain the phenomena"
- You can agree or not → But, although land spectacularly on Titan → still cannot foresee if one of a thousand asteroids will someday end up hitting the Earth!

Titan Ballute

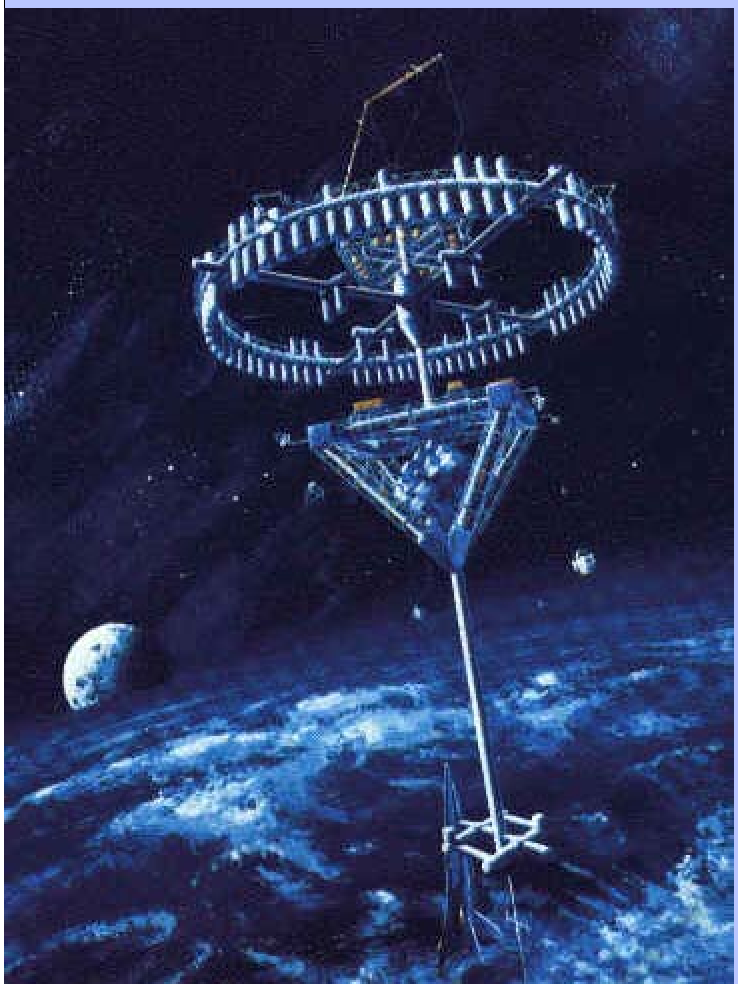


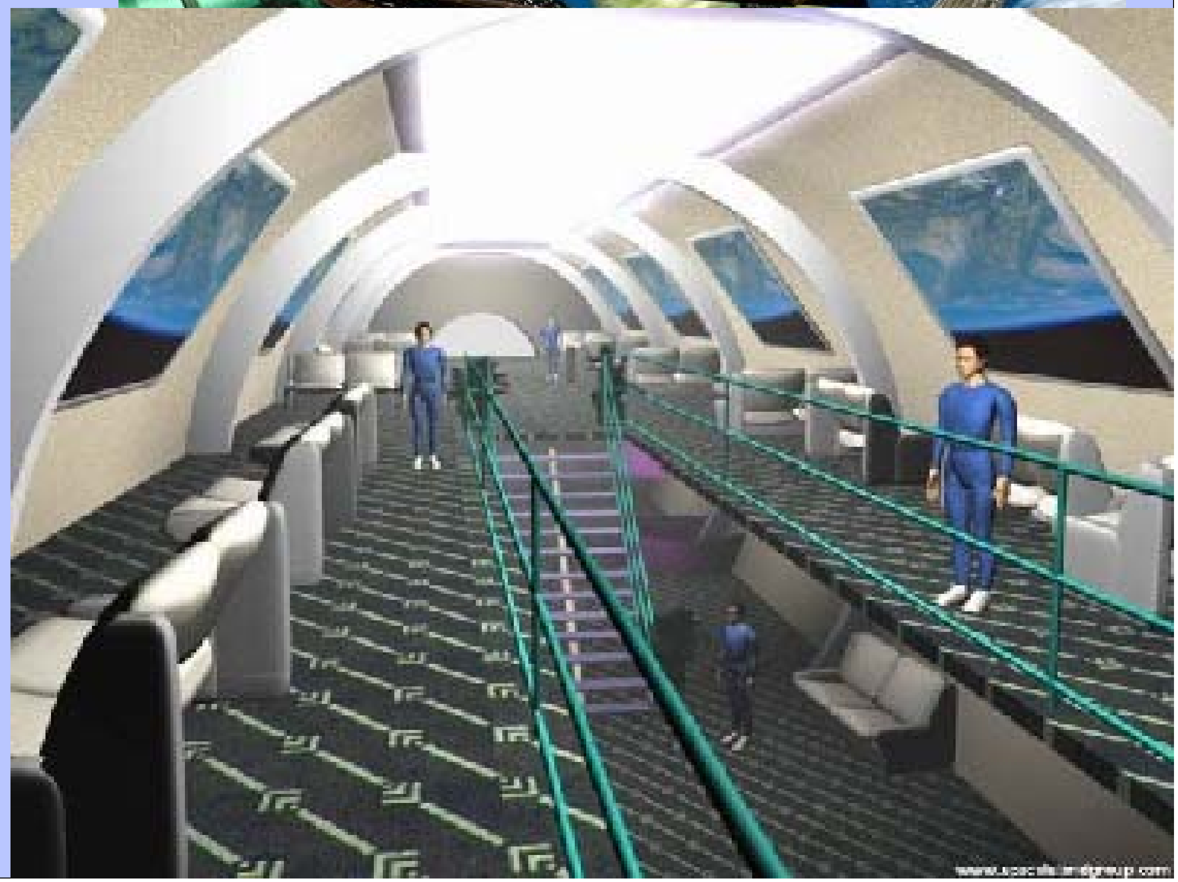
Terrestrial Planet Finder


Joint NASA/ESA
Jupiter Europa + Jupiter Ganymede
JOI 2025

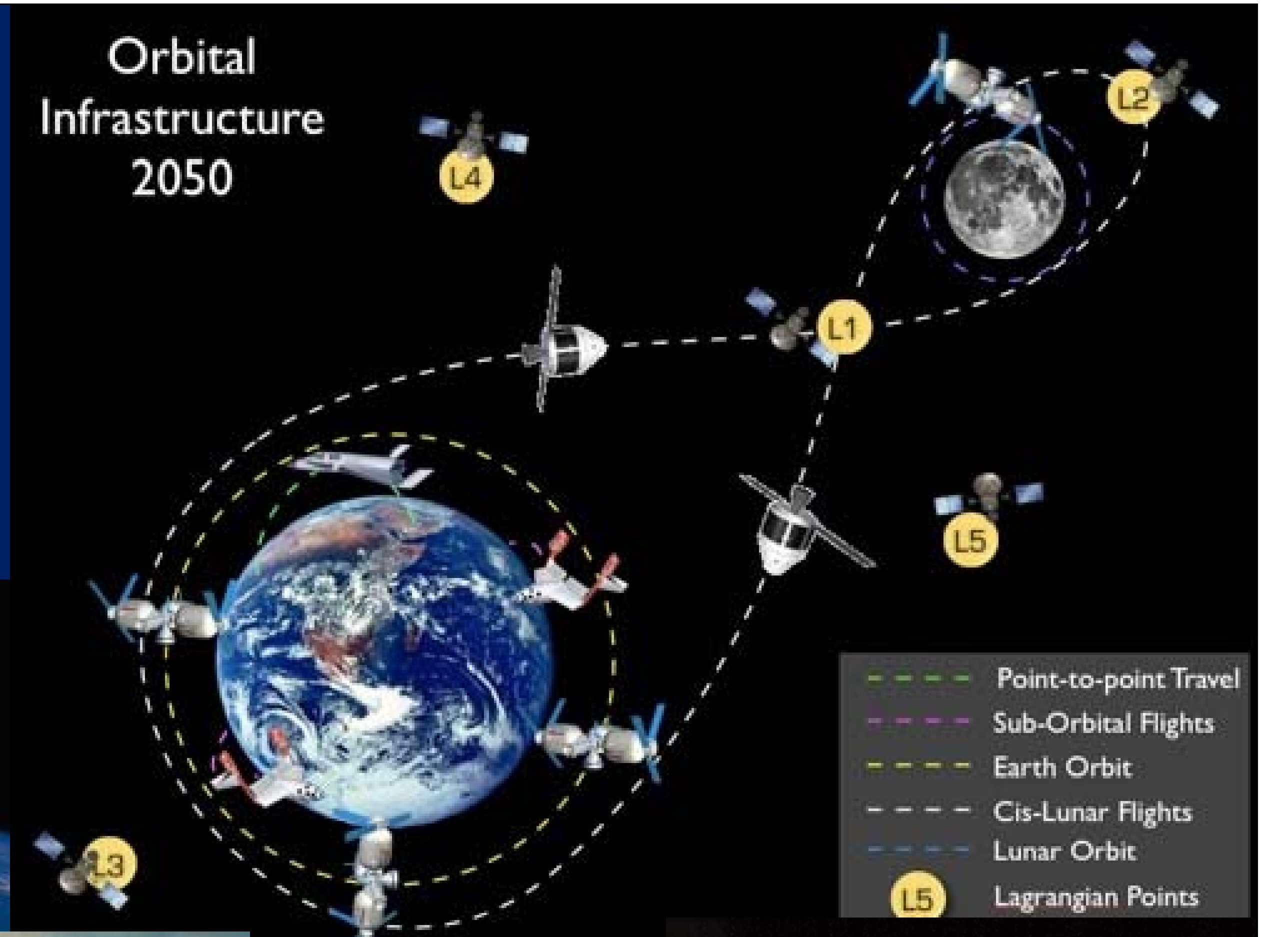

Interstellar Vehicles
2037 ??

Space Tourism and Hotels



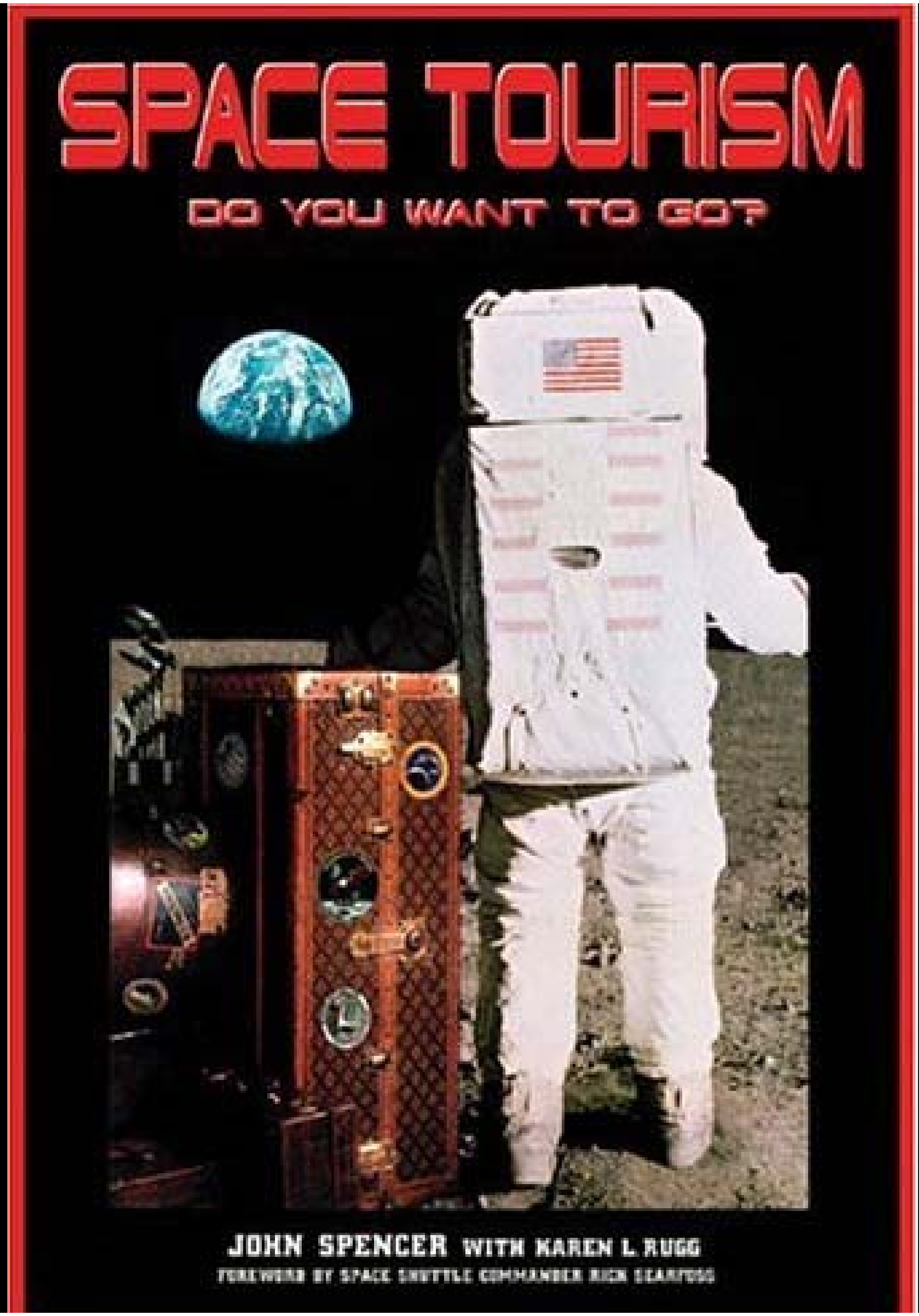



UT
CO
OK
AZ
TX
New Mexico
SPACEPORT
Farmington
Raton
Taos
Gallup
Santa Fe
Albuquerque
Tucumcari
Clovis
Socorro
Truth or Consequences
Roswell
Alamogordo
Silver City
Artesia
Hobbs
Lordsburg
Las Cruces
Carlsbad
Deming
El Paso, Tx.
Mexico


Orbital
Infrastructure
2050
L1
L2
L3
L4
L5
Point-to-point Travel
Sub-Orbital Flights
Earth Orbit
Cis-Lunar Flights
Lunar Orbit
Lagrangian Points

Future?


GALACTIC


SPACE TOURISM
DO YOU WANT TO GO?
JOHN SPENCER WITH KAREN L RUGG