



Fire, Fractals and the Divine Proportion

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1 micron

Talk outline

Beginning stuff

Middle stuff

Ending stuff

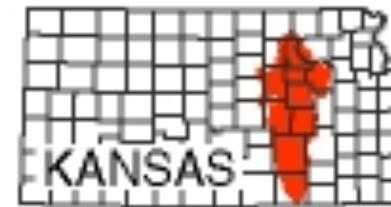
Applause (I hope)

Questions

The Flint Hills of Kansas



wikipedia



Fire Preserves the Prairie



And the prairie blooms again



Small science

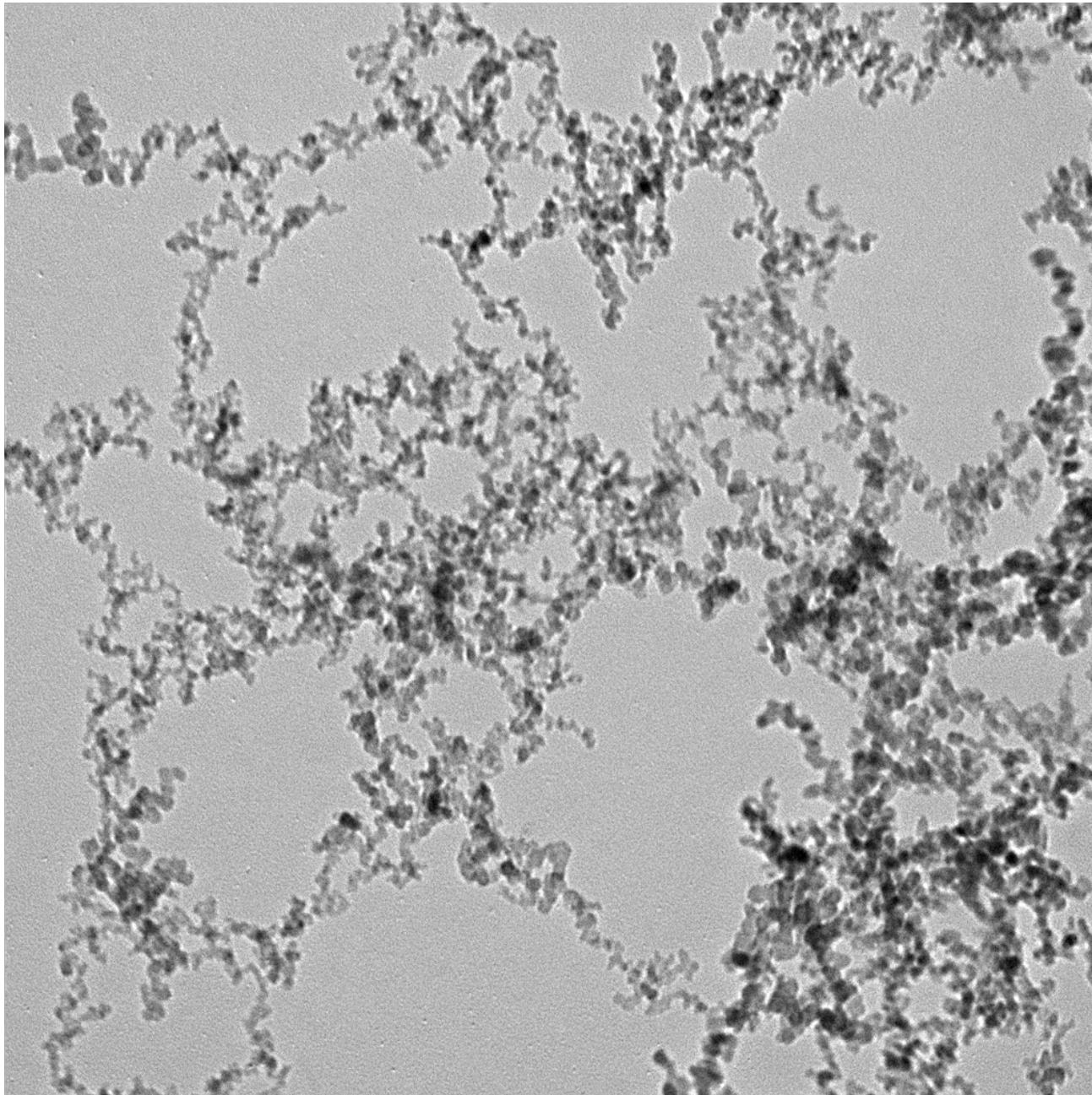
Table top science

One man, with his students,
armed with modest equipment,
seeking out the secrets of Nature!

To see a world in a grain of sand
And a heaven in a wild flower
To hold infinity in the palm of your hand
And eternity in an hour

William Blake
“Auguries of Innocence”





34000 10 min 3.tif

Propane 10 min

Print Mag: 44200x @ 3. in

14:11 07/29/04

100 nm

HV=100kV

Direct Mag: 34500x

A Candle Flame





edf.org



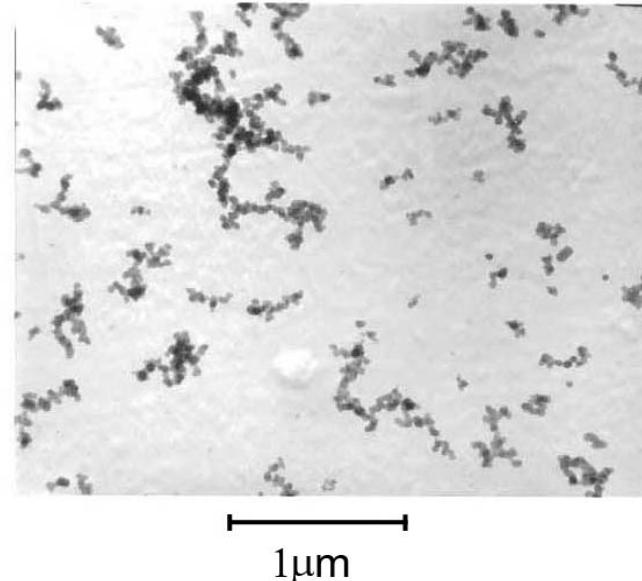
dipity.com

Individual soot
particles captured
from a yellow,
sooting flame.

How do we describe,
indeed, quantify,
their morphology?



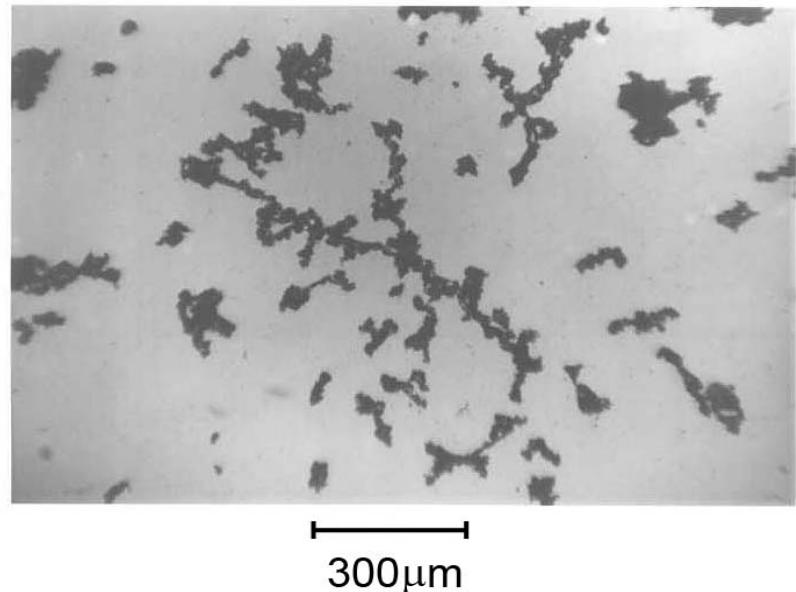
Pictures of soot taken from early in a flame (upper picture) and later in the flame (lower) after the particles grew some more.



Note the similar morphologies.

Then note the scale bars.

Scale Invariance



Sorensen and Feke, AS&T 25, 328 (1996)

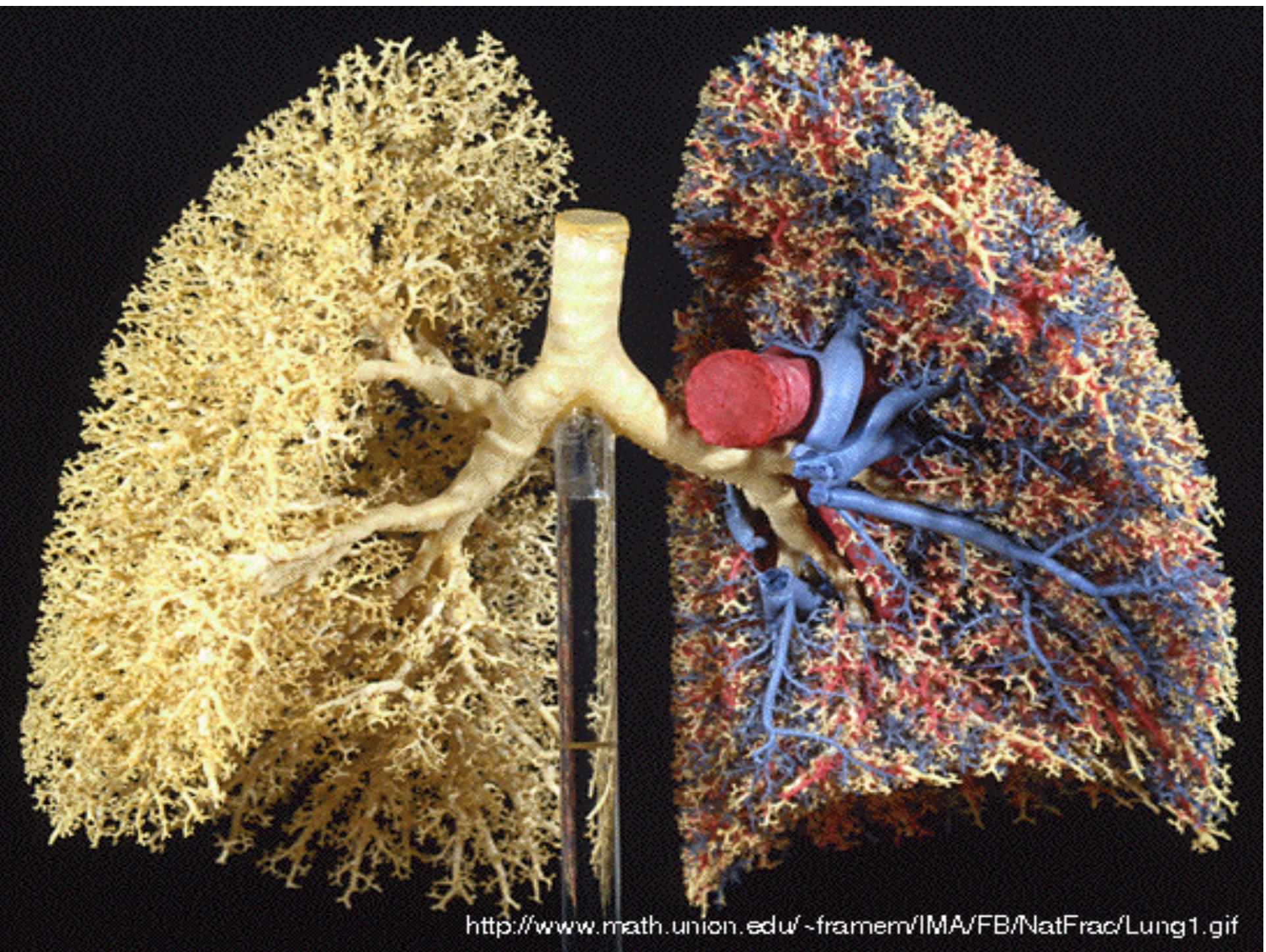
Scale Invariance in Nature





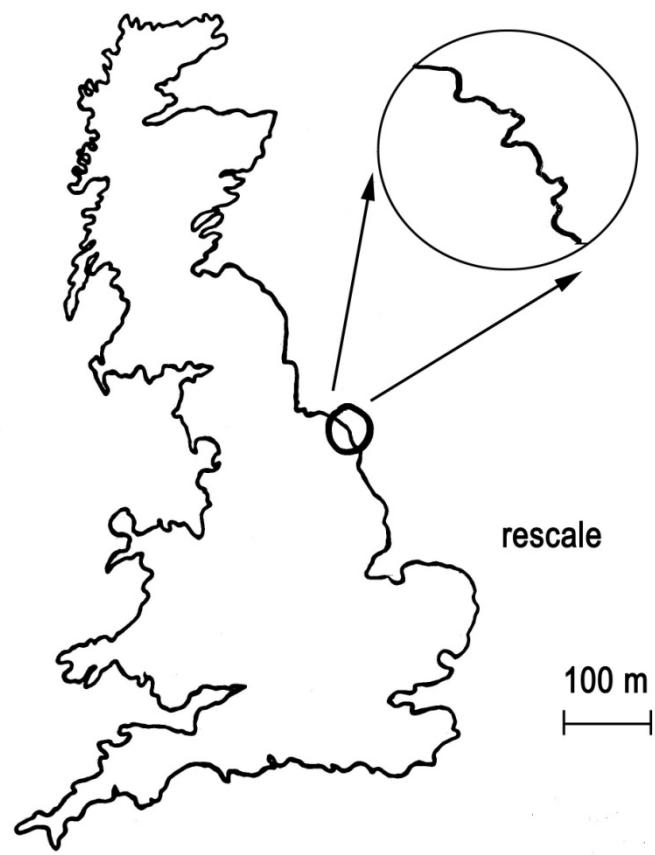


enemyindustry.net



<http://www.math.union.edu/~framen/IMA/FB/NatFrac/Lung1.gif>

Scale Invariance











Scale Invariance

Looking the same regardless of how close or from how far you view it,
i.e. it looks the same regardless of scale.

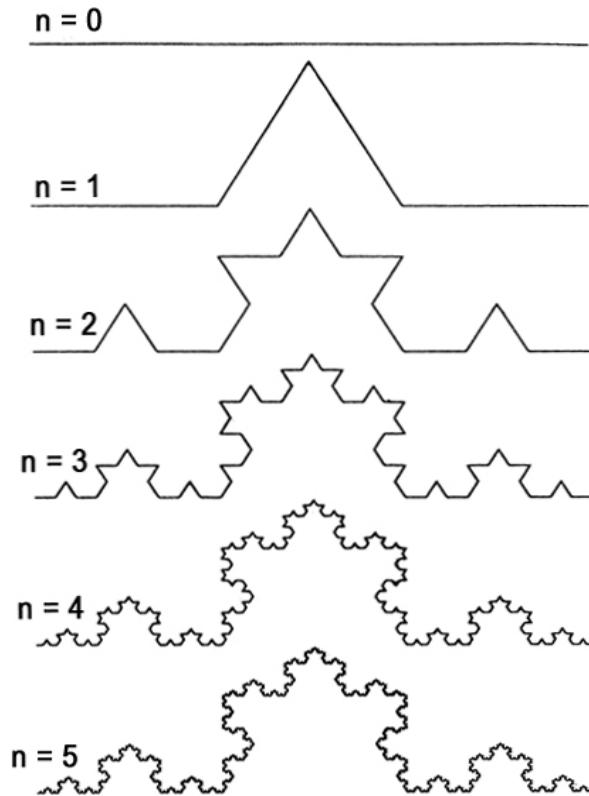
Objects in nature are often scale invariant.
Human-made objects often are not;
they are geometric.

Some scale invariant art



Mathematical Scale Invariance

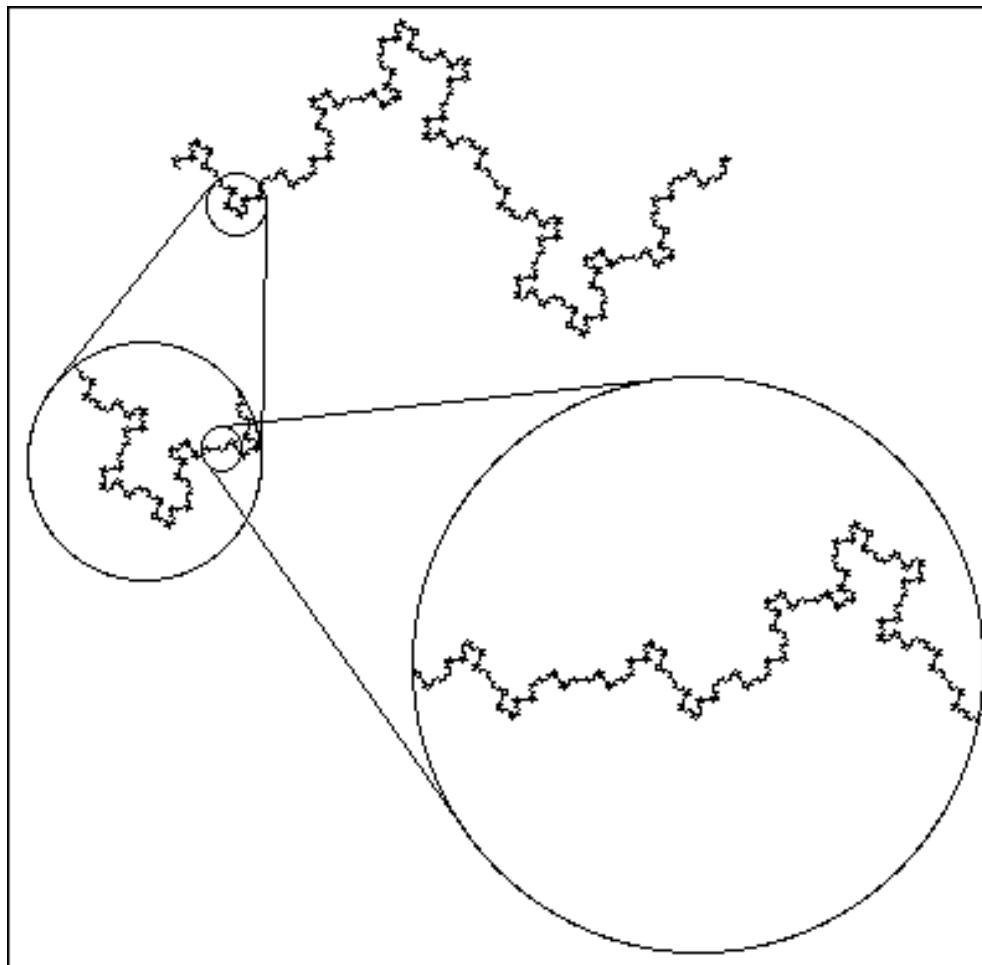
Construction of the Koch curve.



Note how each small section of the lower curves looks like the whole section.
Thus the Koch curve is scale invariant.

Scale Invariance

Change the scale....
see the same thing!



Mandelbrot coined the term
“Fractal”
to name scale invariant objects.

Dimensionality

What would you weigh if you were twice as tall?
(and kept the same proportion)

Twice x twice x twice = 8 times as much.

$2^D = 8$, $D = 3$; D is the scaling dimension.

- The relation $\text{Mass} \sim \text{size}^D$,
is called a “power law”.

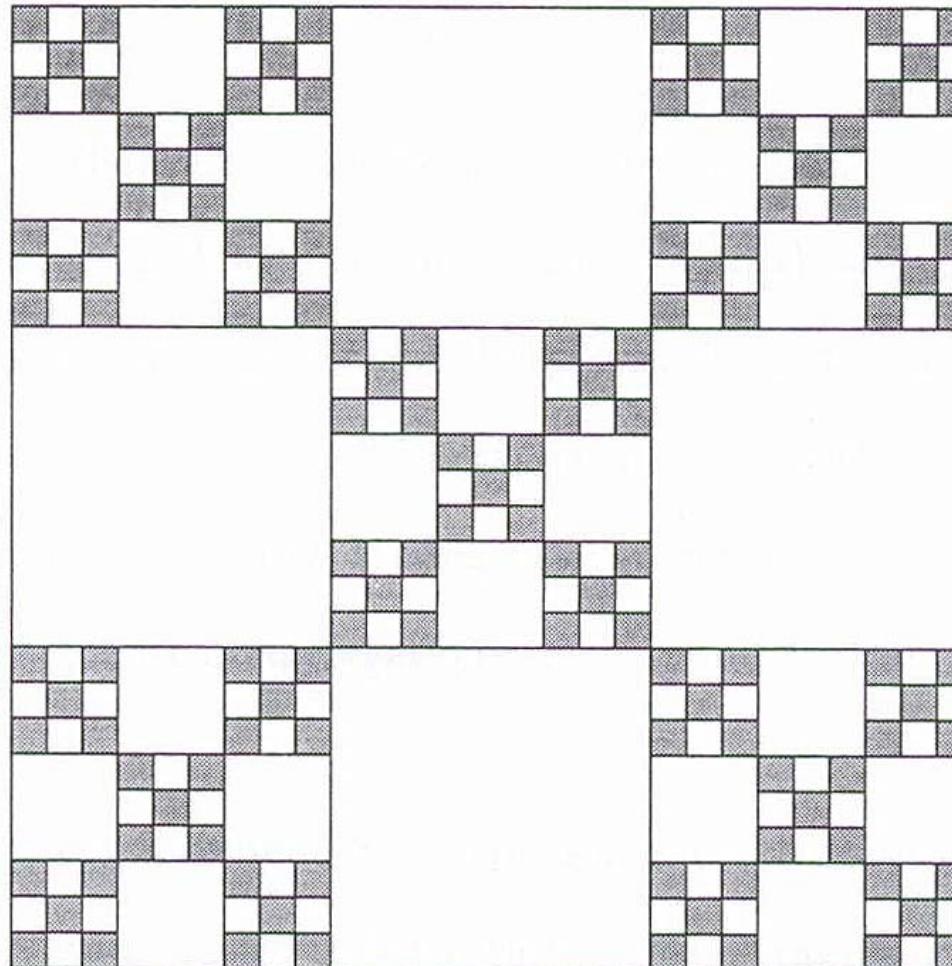
If you were twice as tall...
how much more skin would you have?

Skin is a two-dimensional thing

Two dimensions so two “twices”
twice x twice = 4 times as much

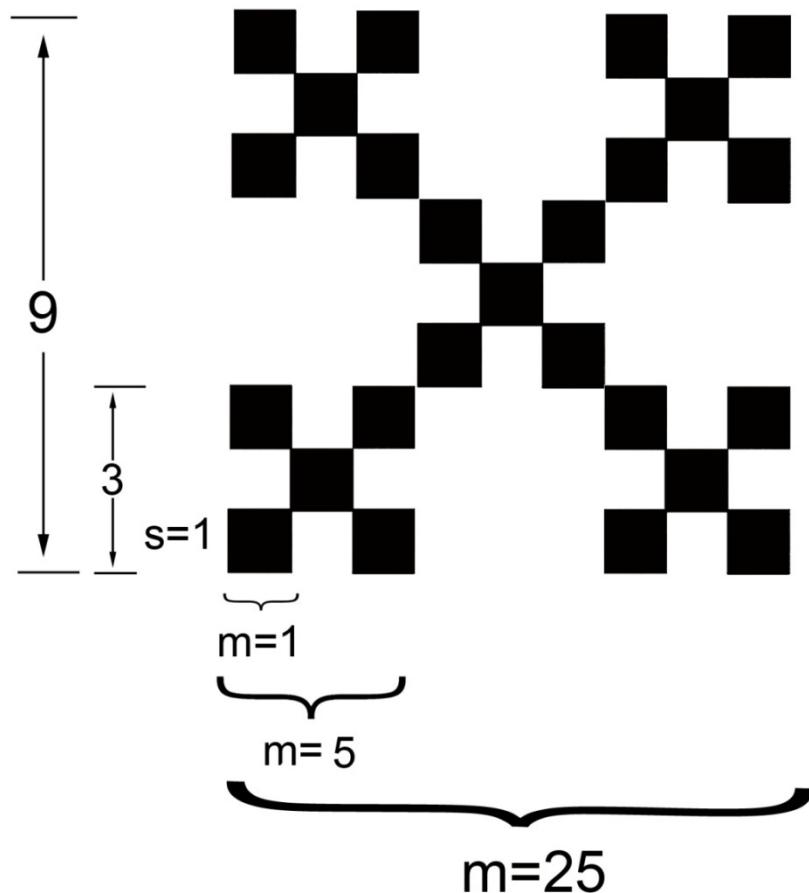
Another power law... 2^D
... different dimension, D.

A simple geometrical fractal.



Ad infinitum

Scaling of a Fractal



Since $m=1$ when $s=1$

$$m=s^D$$

regardless of D . But what must D be to satisfy this equation for $5=3^D$ and $25=9^D$?

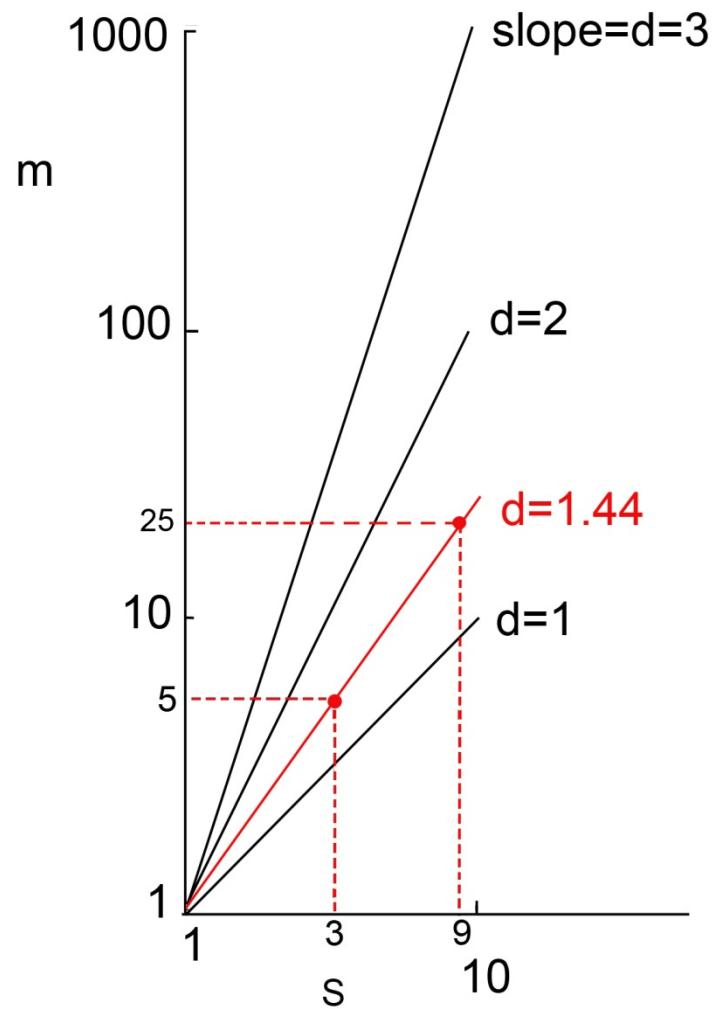
Use logarithms

$$\begin{aligned} D &= \log(5)/\log(3) \\ &= 1.465 \dots \end{aligned}$$

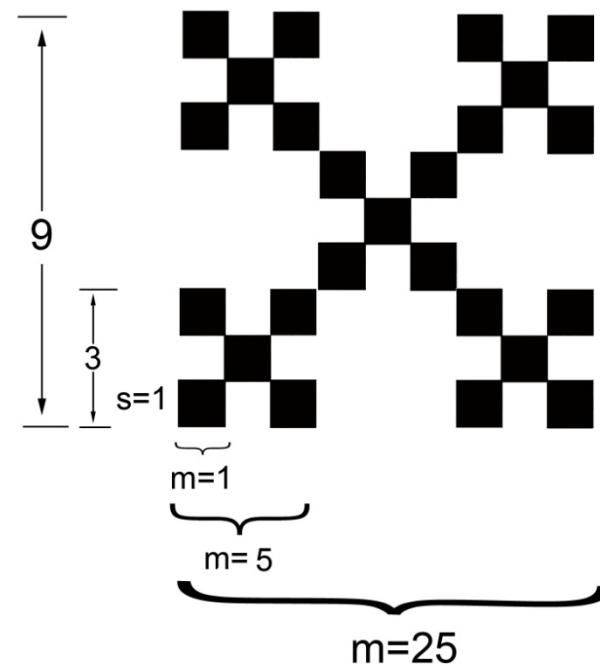
Fractals have non-integer
scaling dimensions!

We will call this scaling dimension
the Fractal Dimension.

Graphical Method



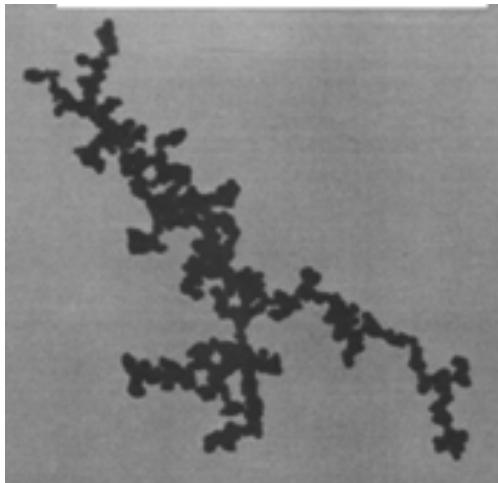
Slope of log-log graph is the dimensionality, D .



Soot fractal aggregates



The fundamental fractal aggregate scaling relation

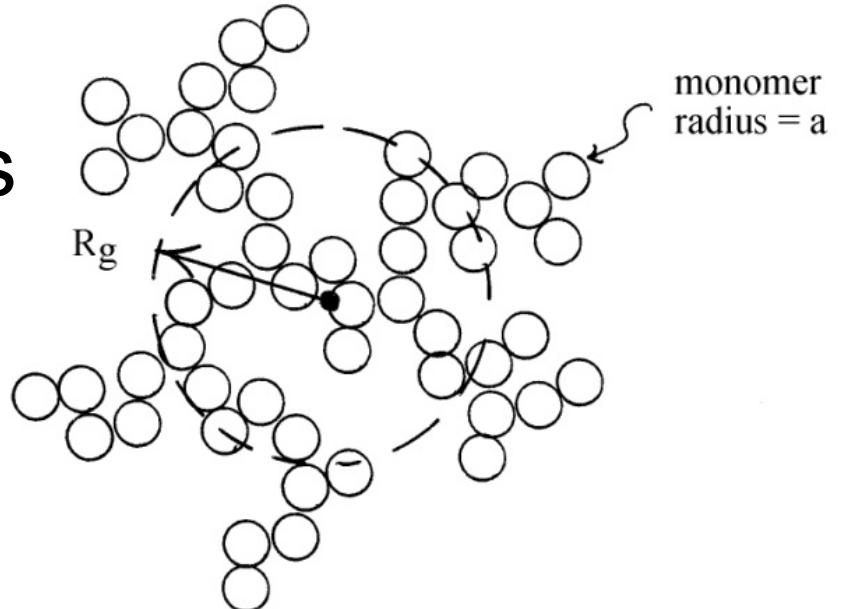


$$N \sim R_g^{D_f}$$

N = number of monomers

R_g = Radius of gyration
a root-mean-square
radius

D_f = fractal dimension



Ensemble Analysis: Flame Soot

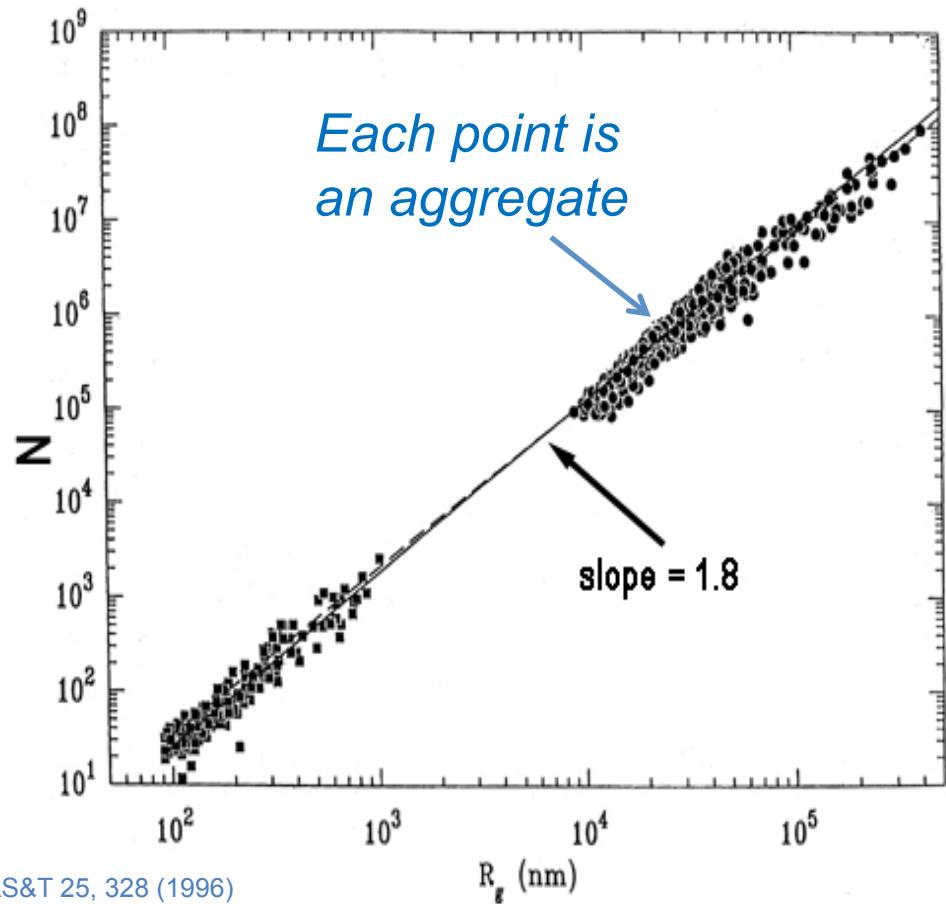
Each cluster of an ensemble has an N and a R_g ,
a single point on this graph.

The log-log plot uncovers
the power law

$$N \sim R_g^{Df}$$

with

$$D_f \approx 1.8$$



Sorensen and Feke AS&T 25, 328 (1996)

With systematic, quantitative observation,
nature reveals a secret

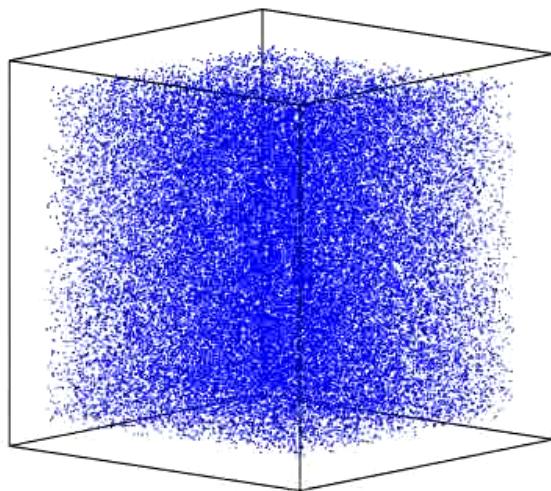
All aggregates, whether from aerosols or colloids,
regardless of chemical composition,
yield the same fractal dimension!

$$D_f \approx 1.8$$

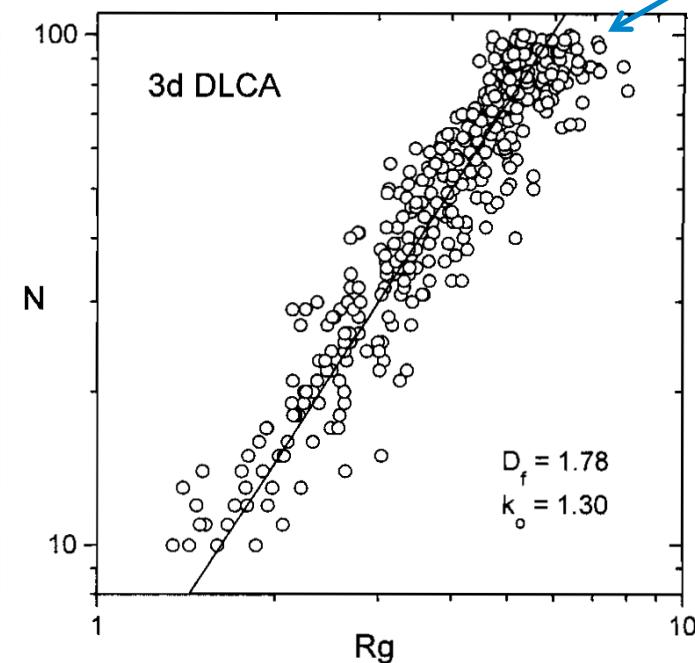
The aggregates (i.e. clusters) move randomly
via diffusion to meet, stick together and aggregate.
This is called Diffusion Limited Cluster Aggregation,
DLCA for short.

DLCA simulation.
40,000 monomers, volume fraction 2×10^{-2}

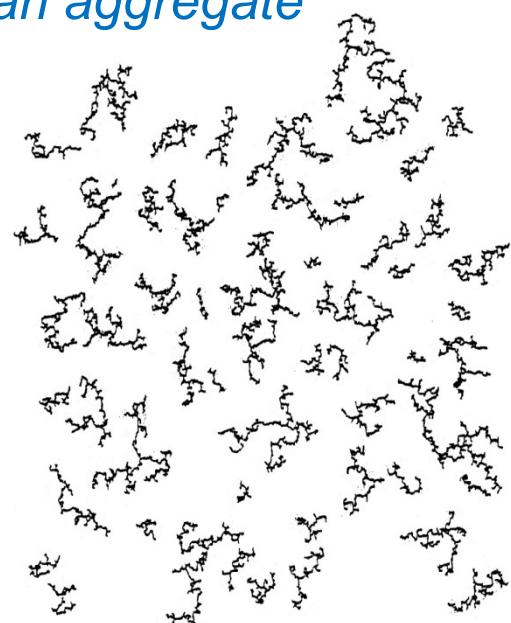
Time = 10



Computer Simulation of Diffusion Limited Cluster Aggregation DLCA

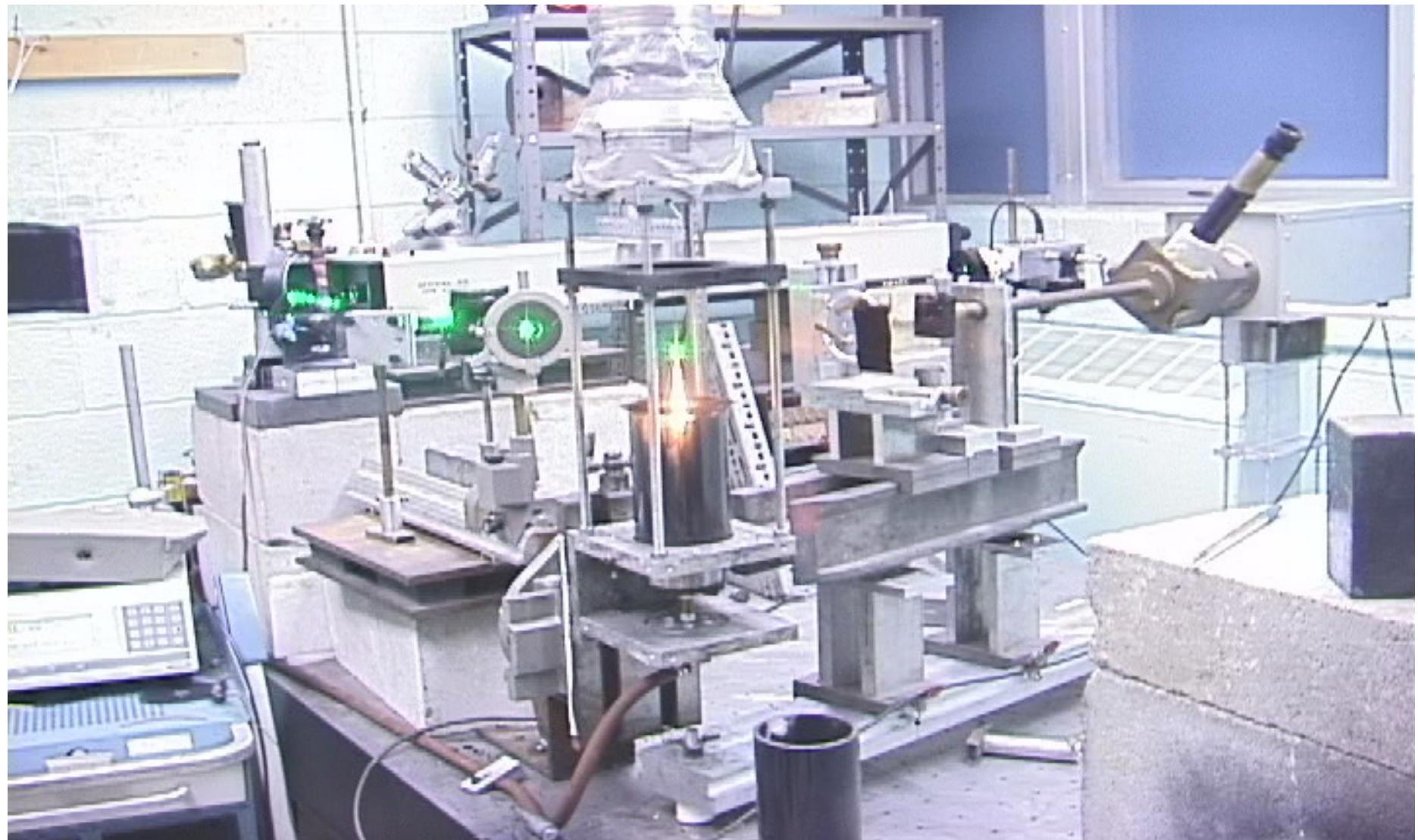


*Each point is
an aggregate*



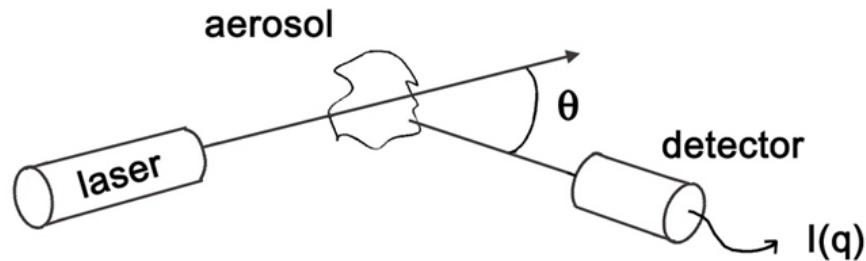
$$\ln 3d, D_f = 1.8 \pm 0.1, k_0 = 1.3 \pm 0.2$$

Light Scattering



Light Scattering Analysis

Measure the scattered light intensity as a function of the scattering angle θ , $I(\theta)$.

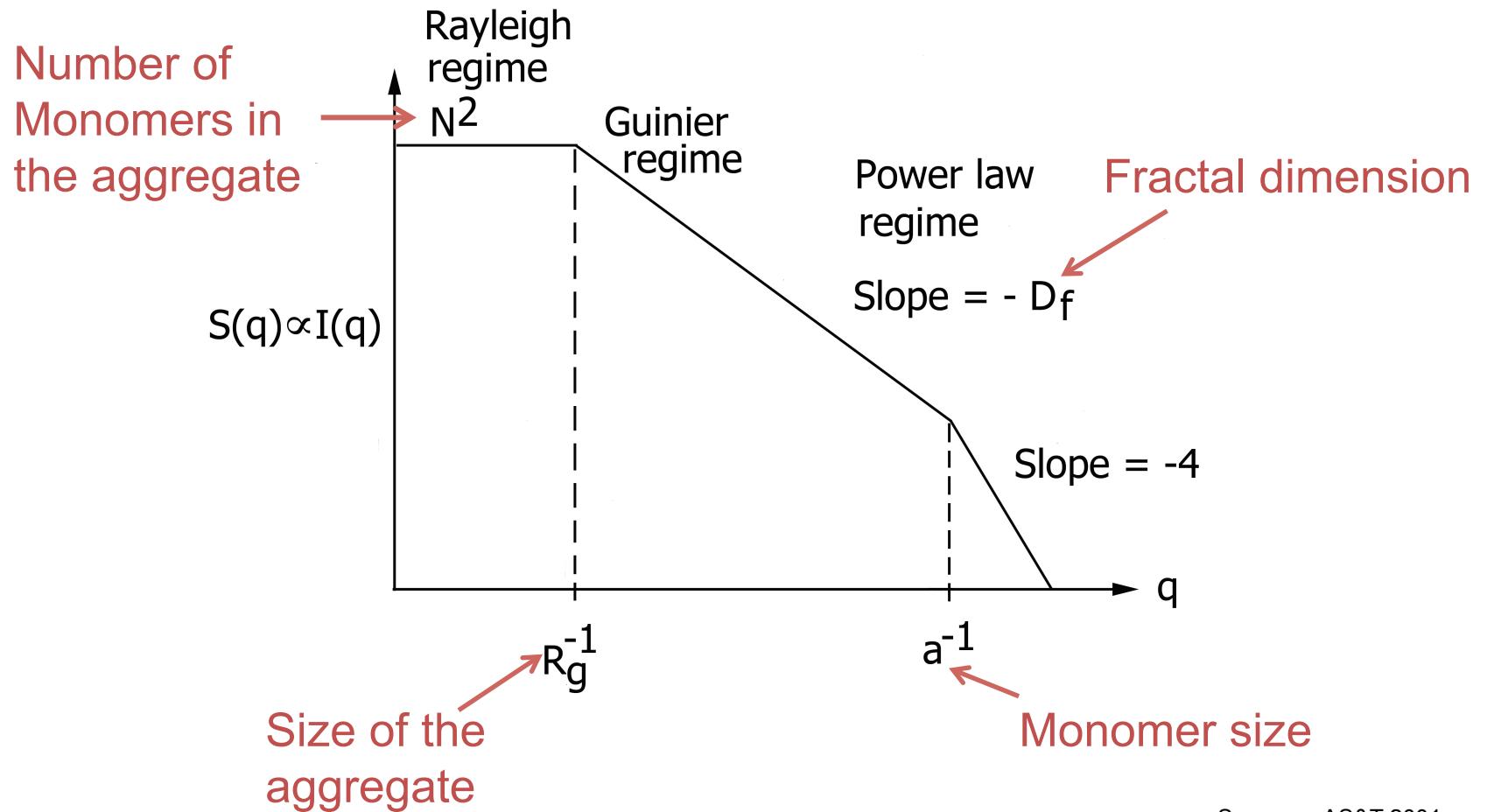


Convert θ and the light wavelength λ to the scattering wave vector

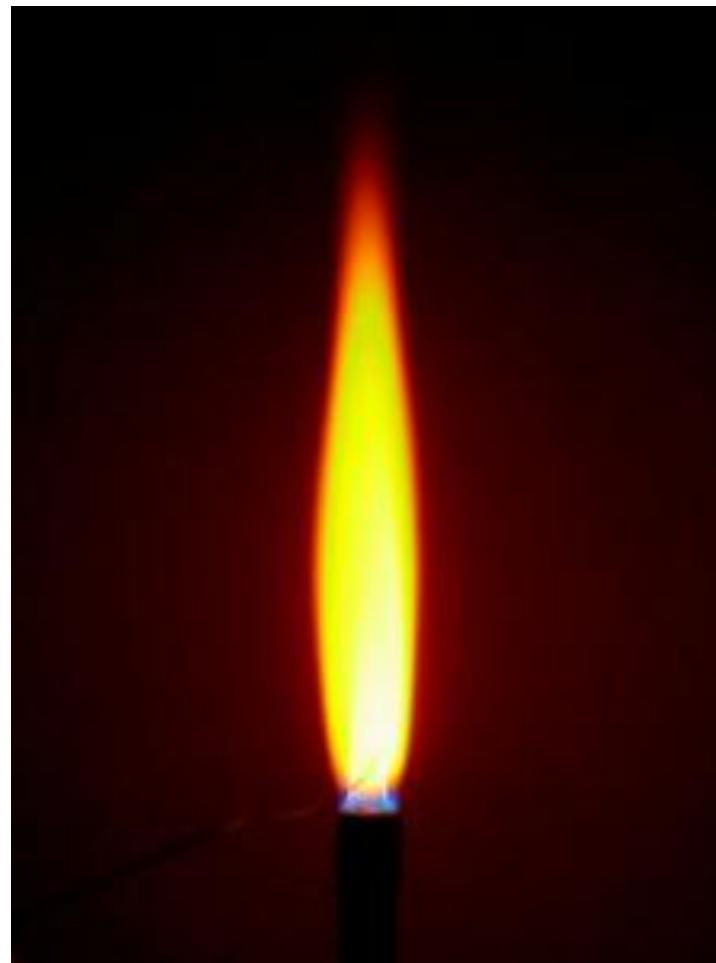
$$q = (4\pi/\lambda) \sin(\theta/2)$$

Then plot $I(q)$ vs q , log-log, to reveal the structure of the scattering particles.

The Fractal Aggregate Structure Factor $I(q)$ or $S(q)$ (log-log plot)

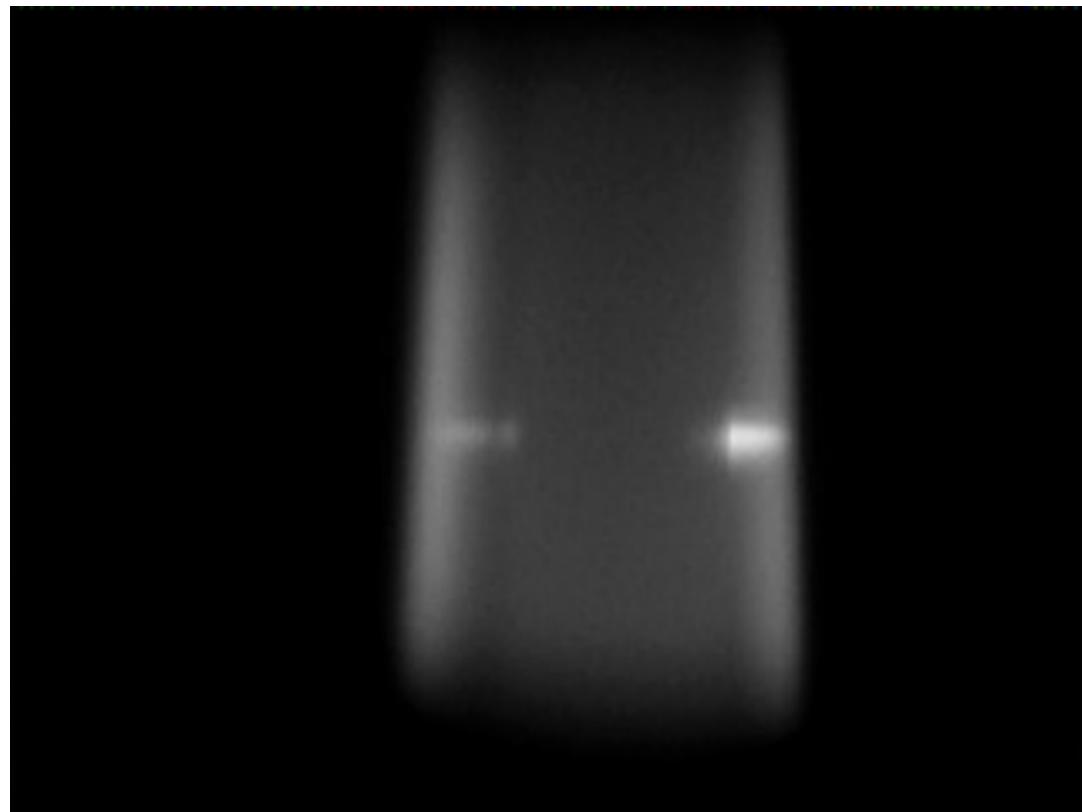


Bunsen burner diffusion flame



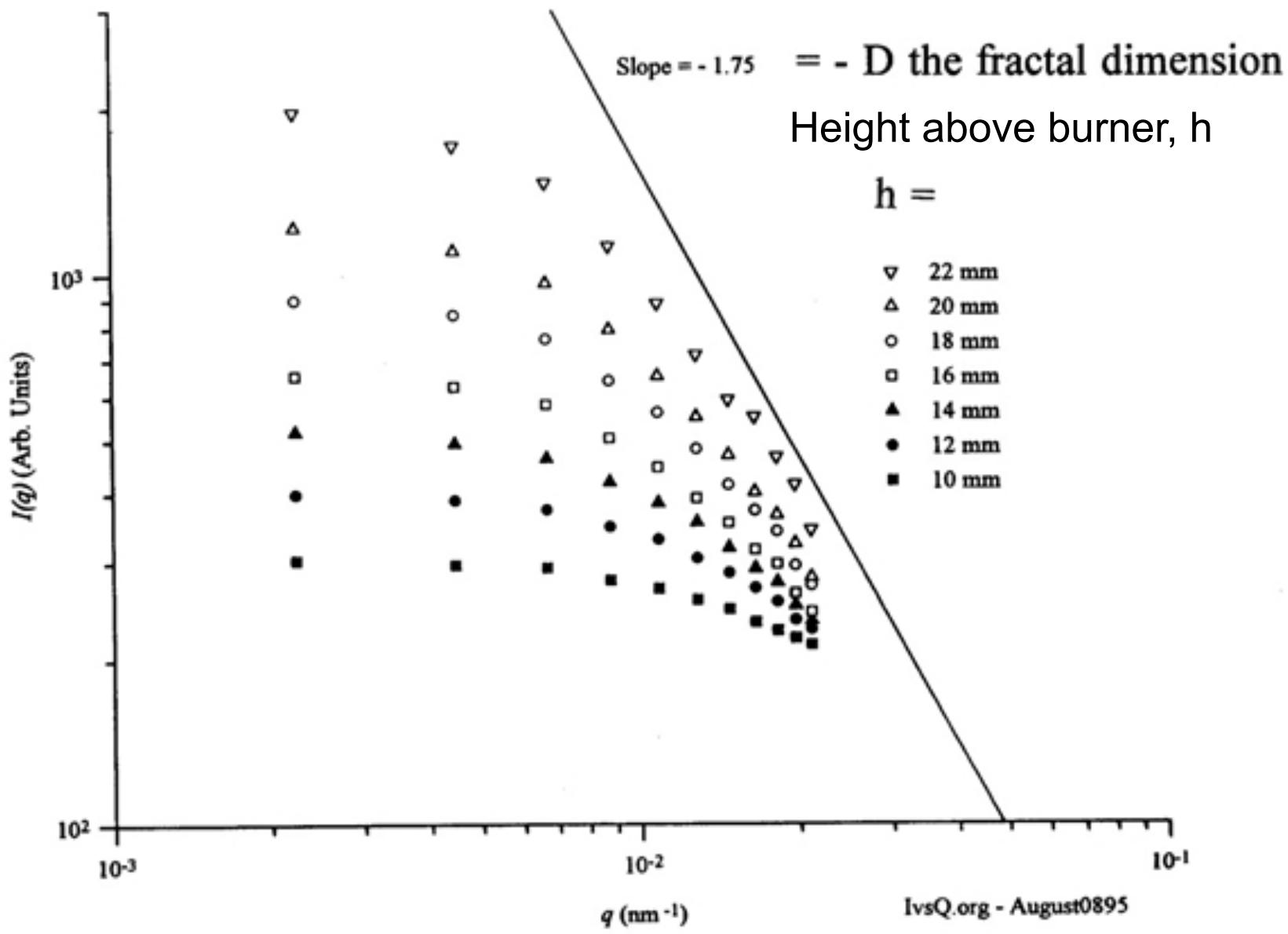
↑
Height above
burner

Laser Beam Passing Through Luminous, Sooting, Diffusion Flame

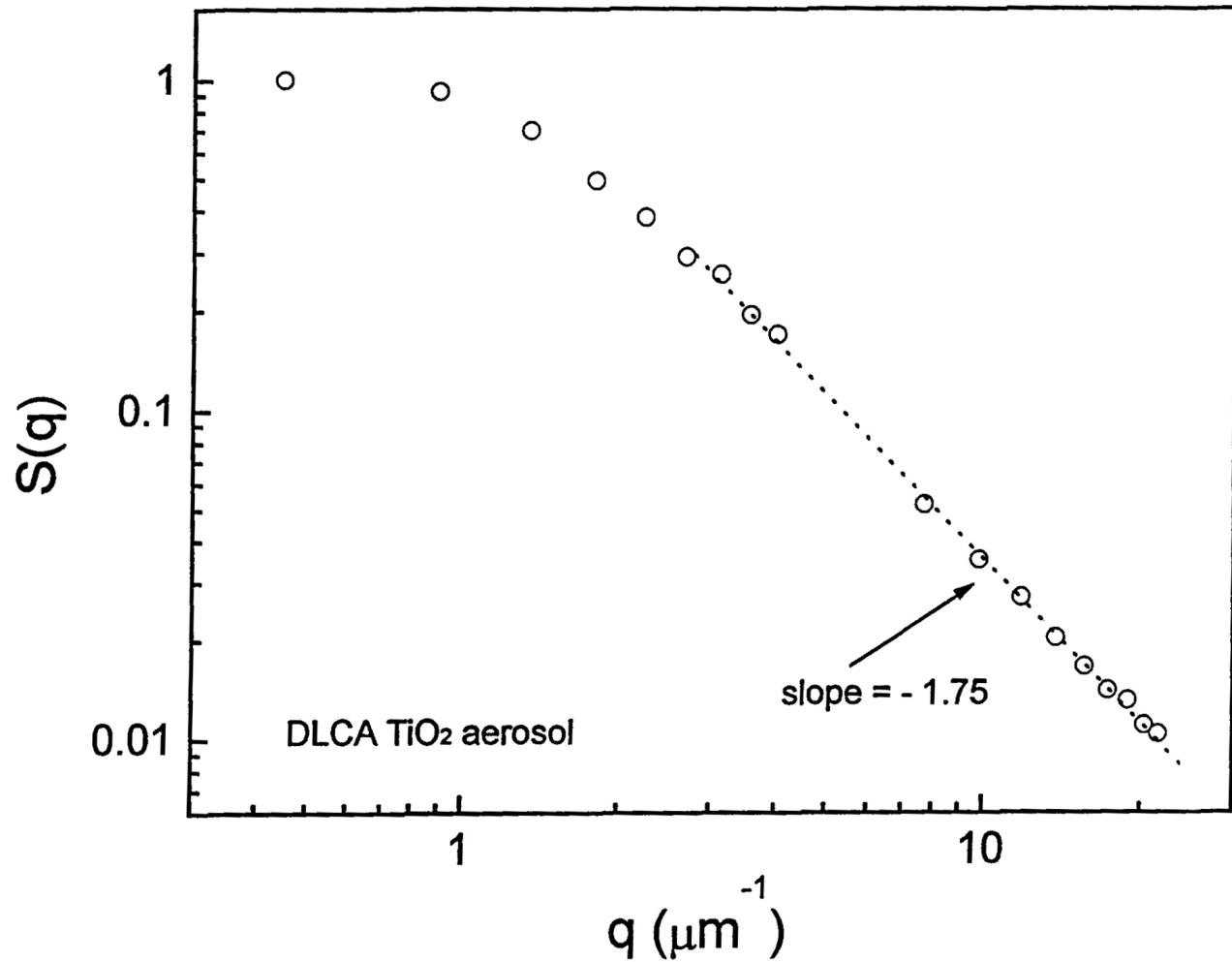


Laser
 $\lambda = 532 \text{ nm}$

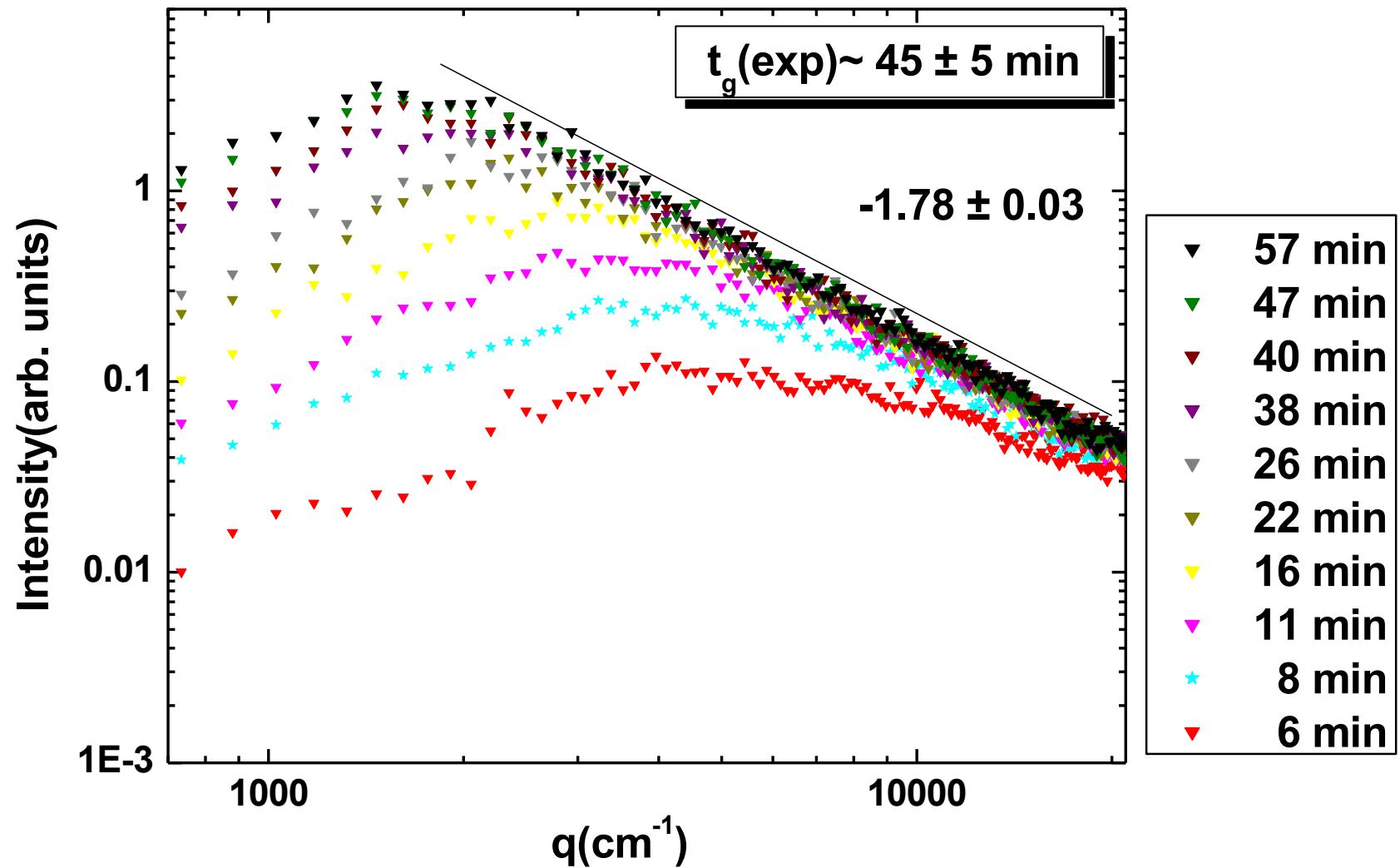
Methane Flame Soot



Structure factor of a TiO_2 aerosol



Colloid Aggregation to a Gel



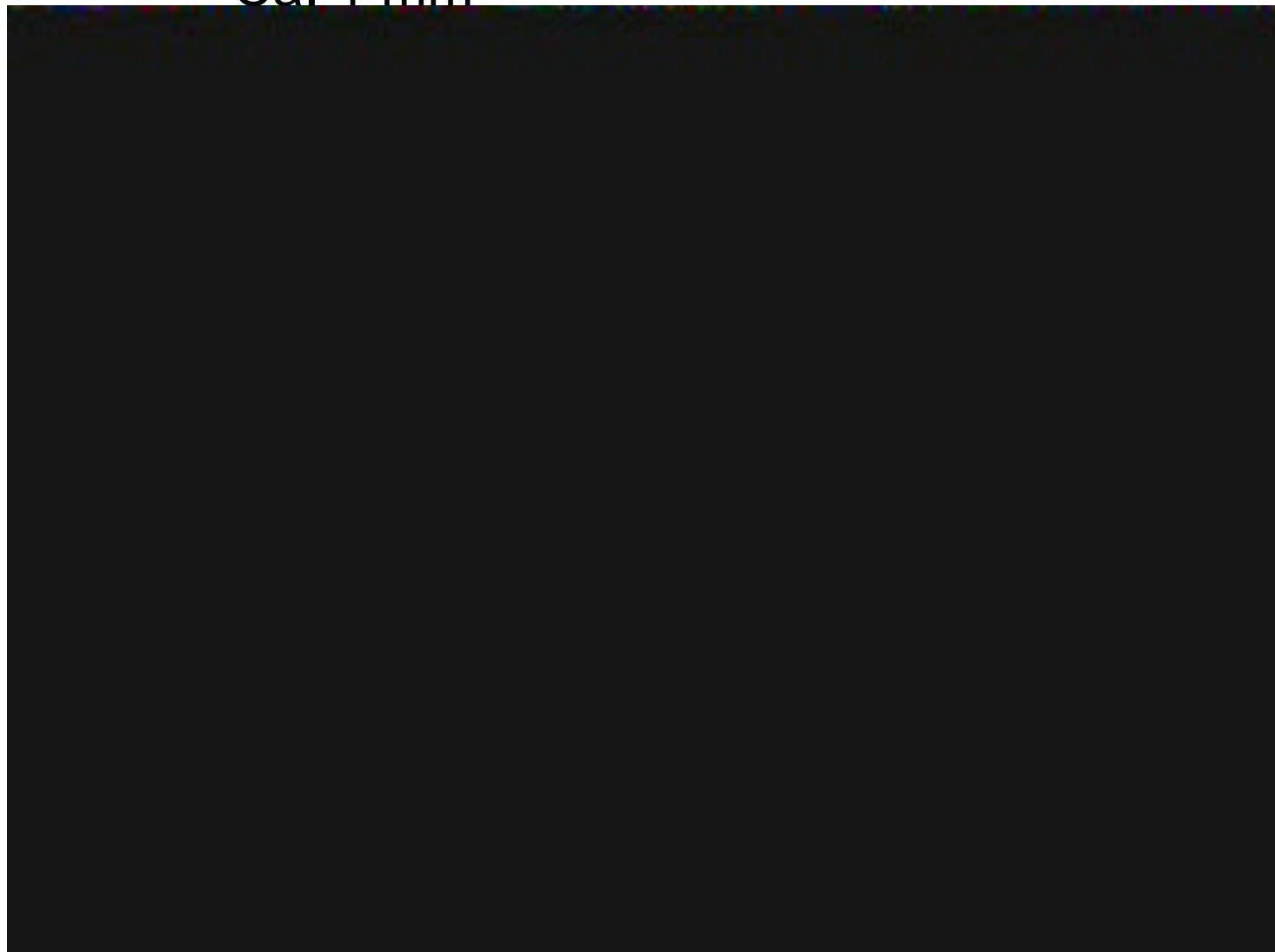
Surprises in the lab!

Video of Acetylene Flame

30 Hz

7 nanosecond exposures

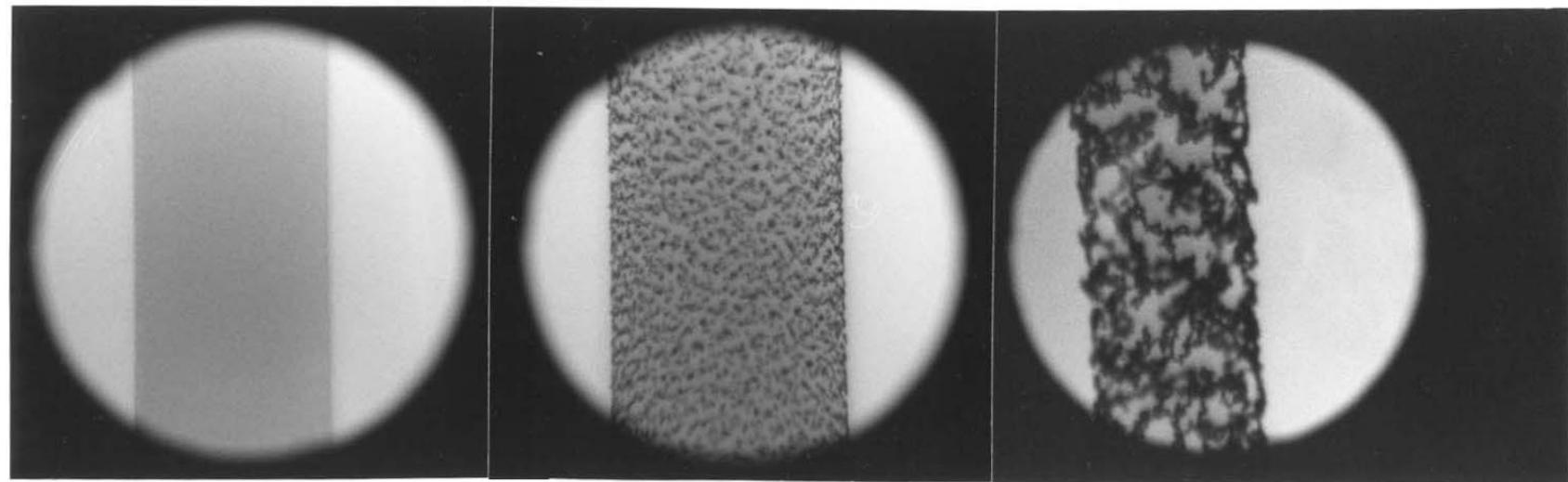
Ca. 1 mm



Aerosol Gelation

Soot in a C_2H_2 Bunsen diffusion flame

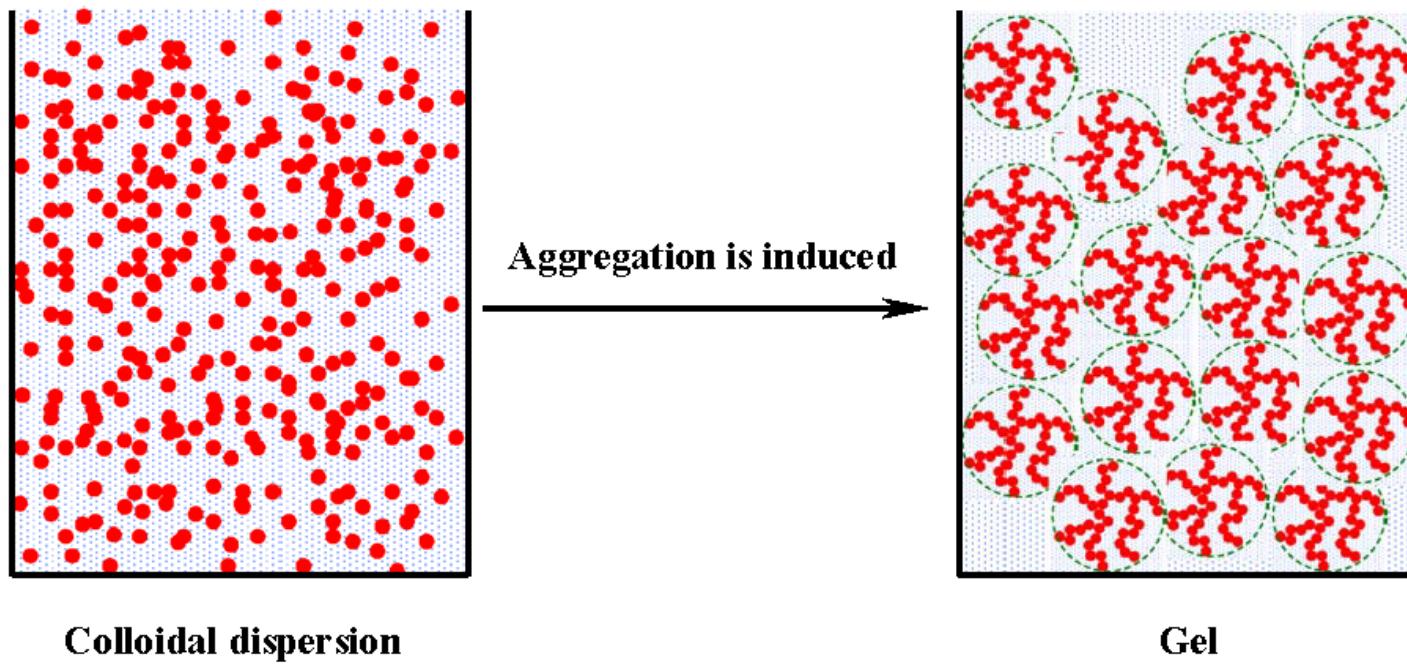
Increasing acetylene →



1mm

b

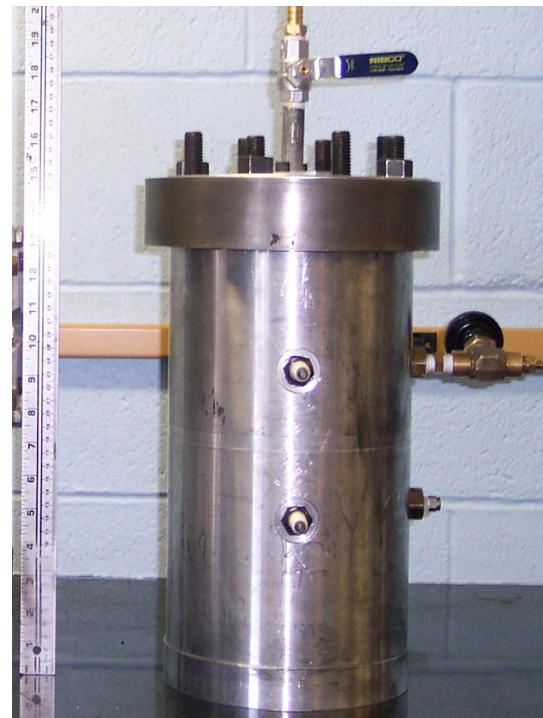
c



The Bombs

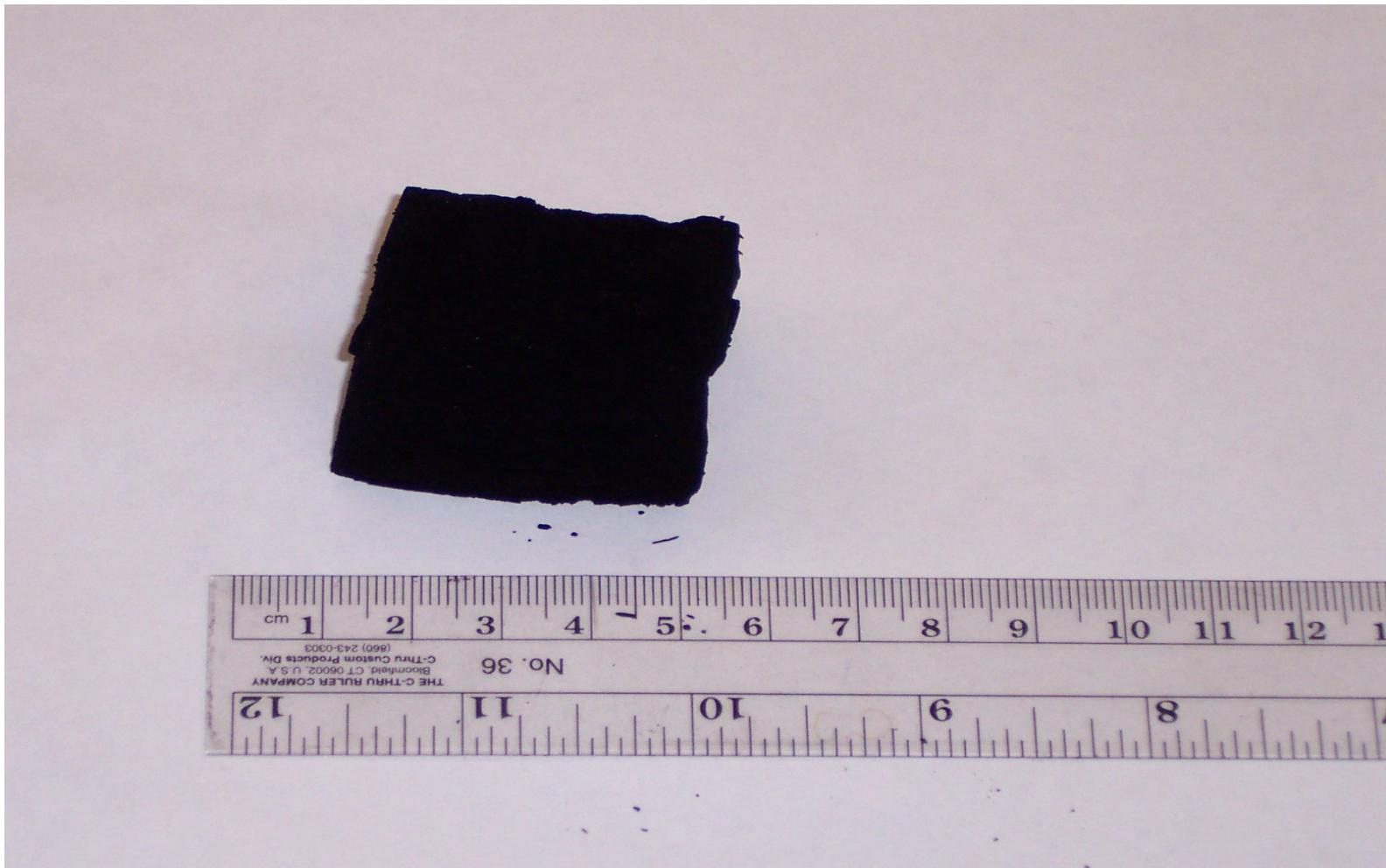


Fat Man, 17 Liters



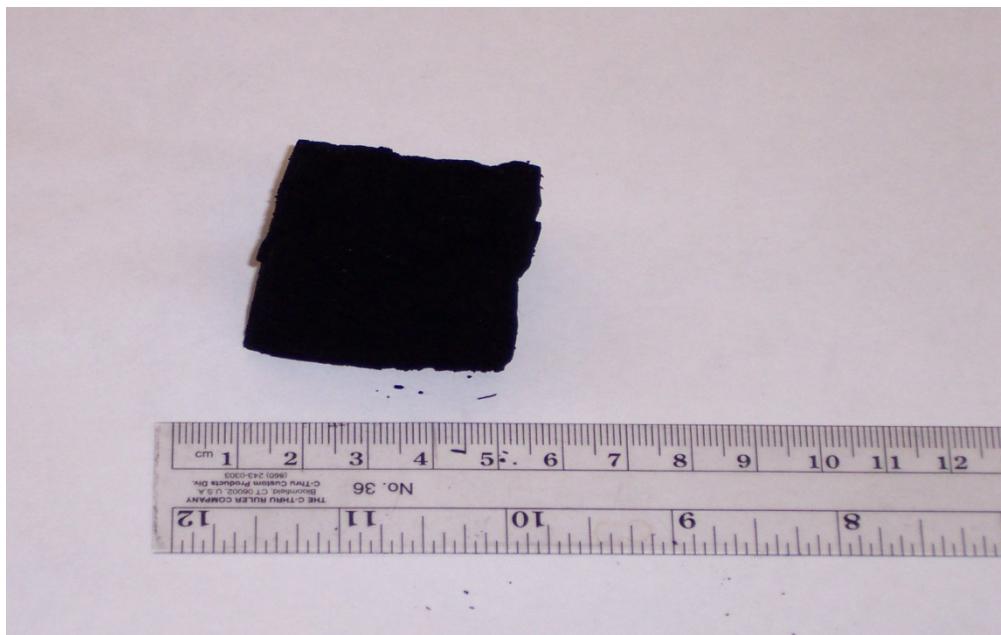
Little Boy, 4 Liters

Carbon aerosol gel density = 2.5 mg/cc.



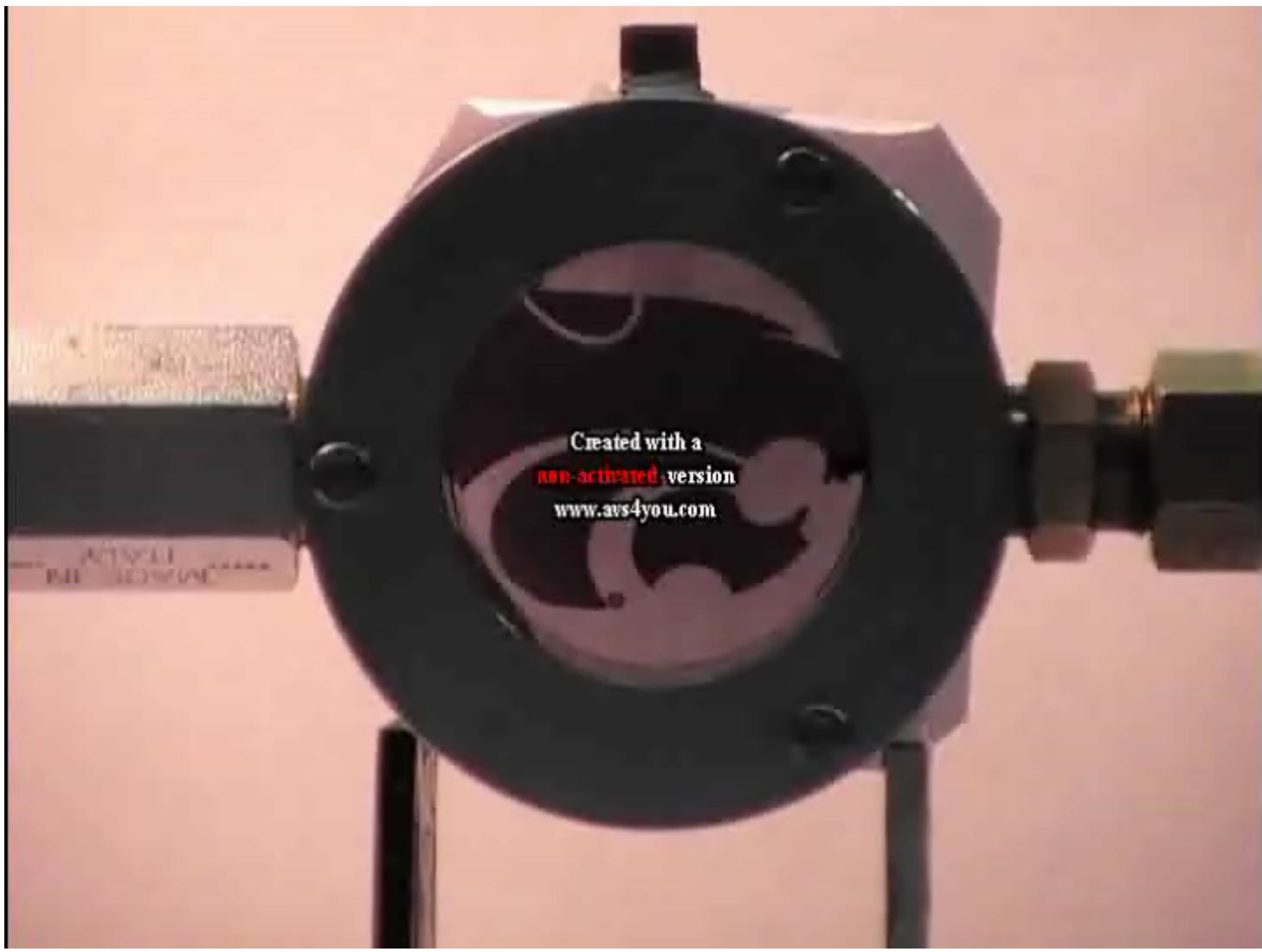


Carbon and silica aerosol gels



Gelation





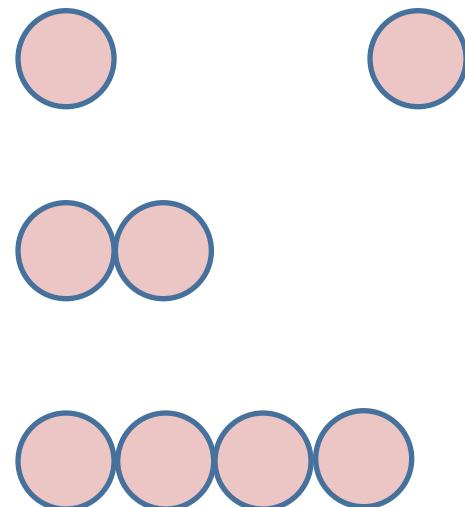
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non-activated version
www.arts4you.com

A simple theory for fractal aggregates

By means of the easy and simple
we grasp the laws of the whole world.

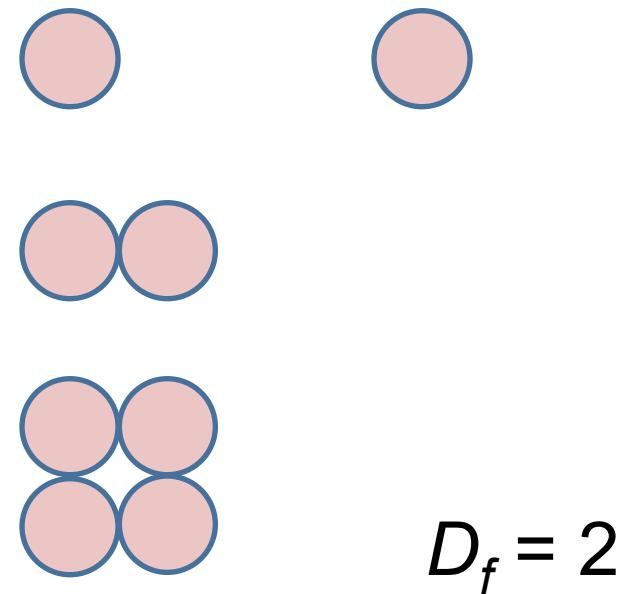
The I Ching

End to end aggregation

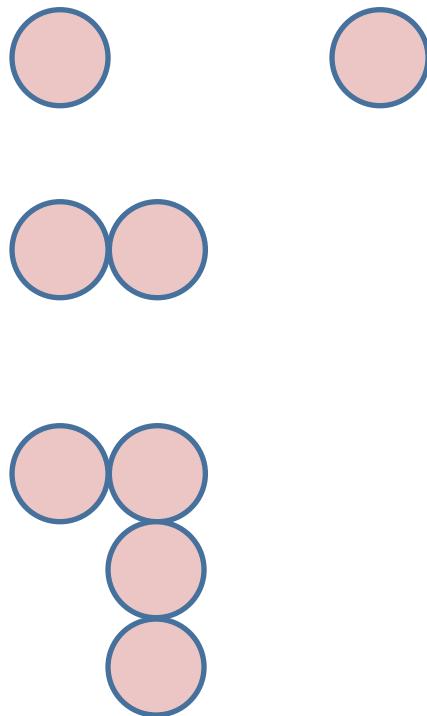


$$D_f = 1$$

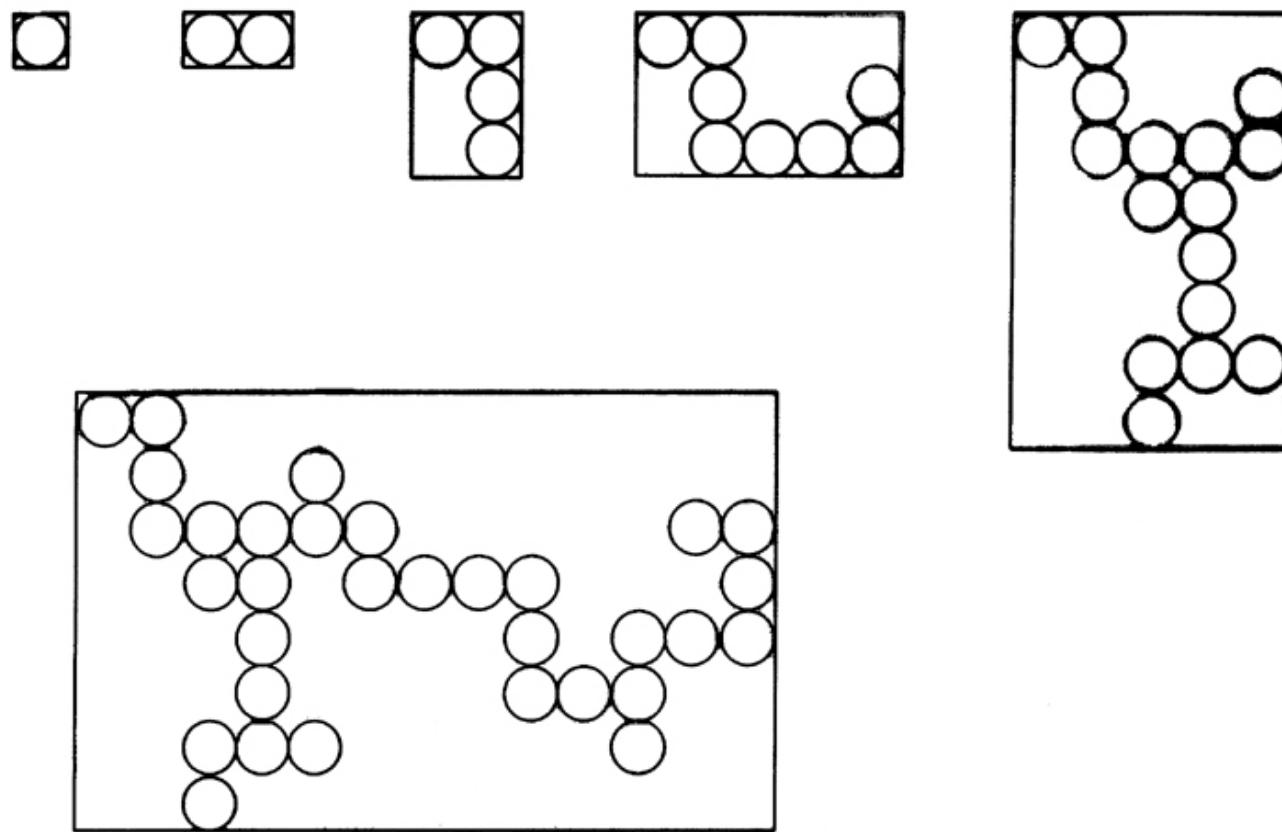
Side to side aggregation



Side to end aggregation



Side-to-end aggregation.



Sorensen and Oh, Phys. Rev. E58, 7545 (1998)

The Fibonacci Series

1, 1, 2, 3, 5, 8, 13, 21, 34, ...

The ratio of successive Fibonacci
Numbers limits to the Divine proportion

$$\phi = \lim_{n \rightarrow \infty} \frac{f_{n+1}}{f_n}$$

$$\Phi = 1.618 \dots$$

Fractal dimension of the model clusters

With each step

N increases by 2

length increases by Φ

So

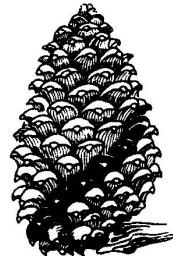
$$2 = \Phi^{D_f}$$

Thus

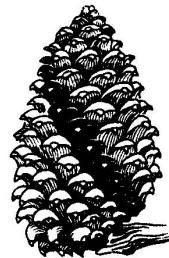
$$\begin{aligned} D_f &= \log(2)/\log(\Phi) \\ &= 1.44\dots \end{aligned}$$

Correct for $d = 2!$

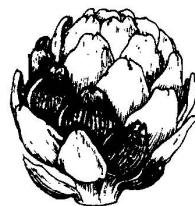
Fibonacci Numbers Abound in Nature



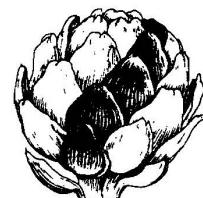
8 parallel rows
spiraling gradually



13 parallel rows
spiraling steeply



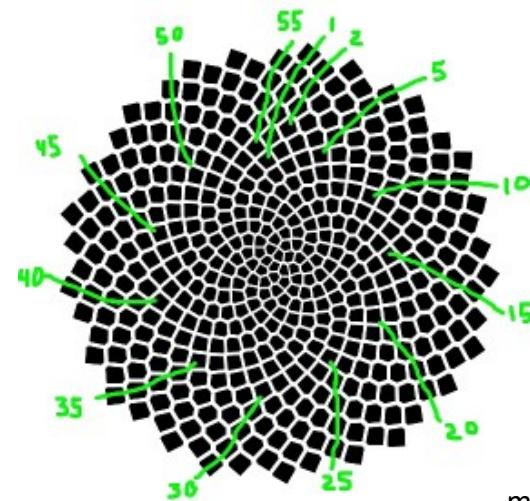
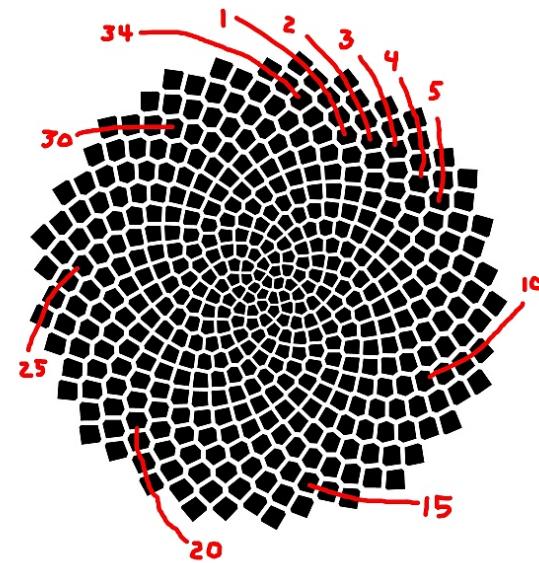
5 parallel rows
spiraling gradually



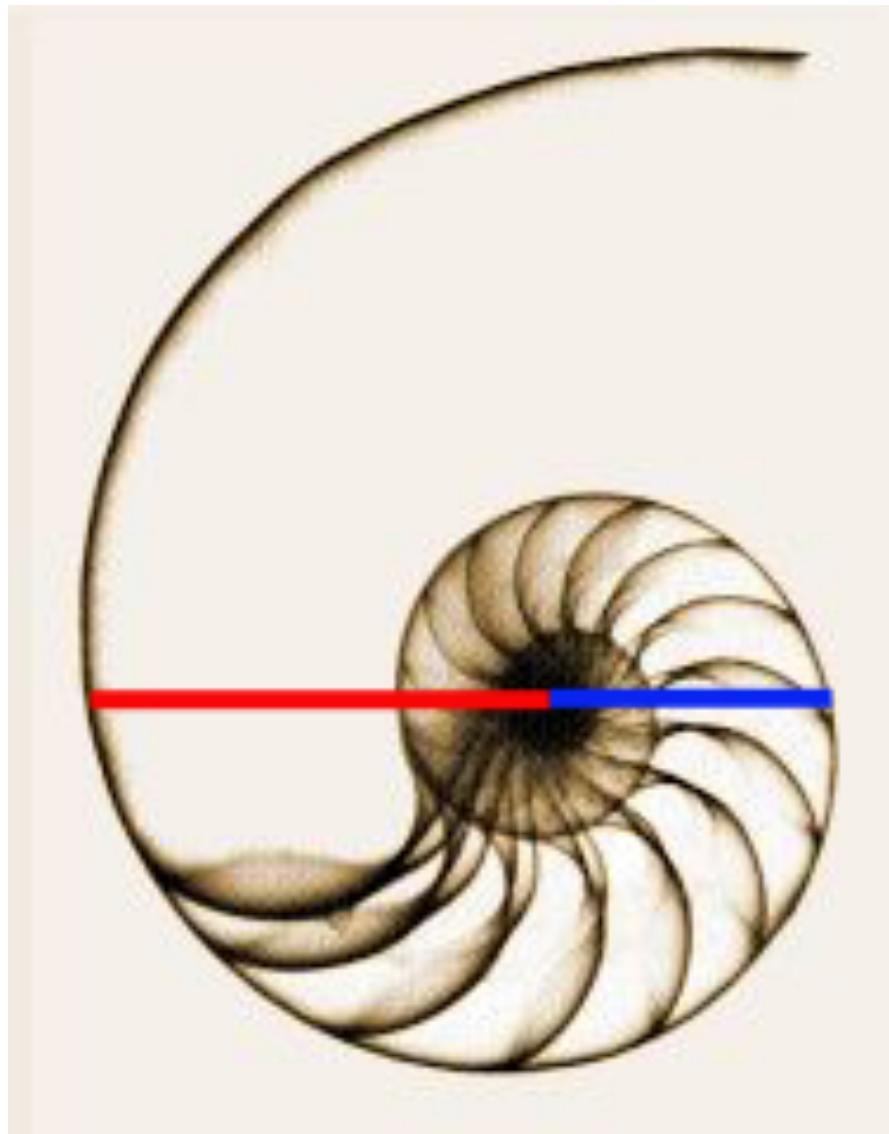
8 parallel rows
spiraling steeply

Sunflower Inflorescence

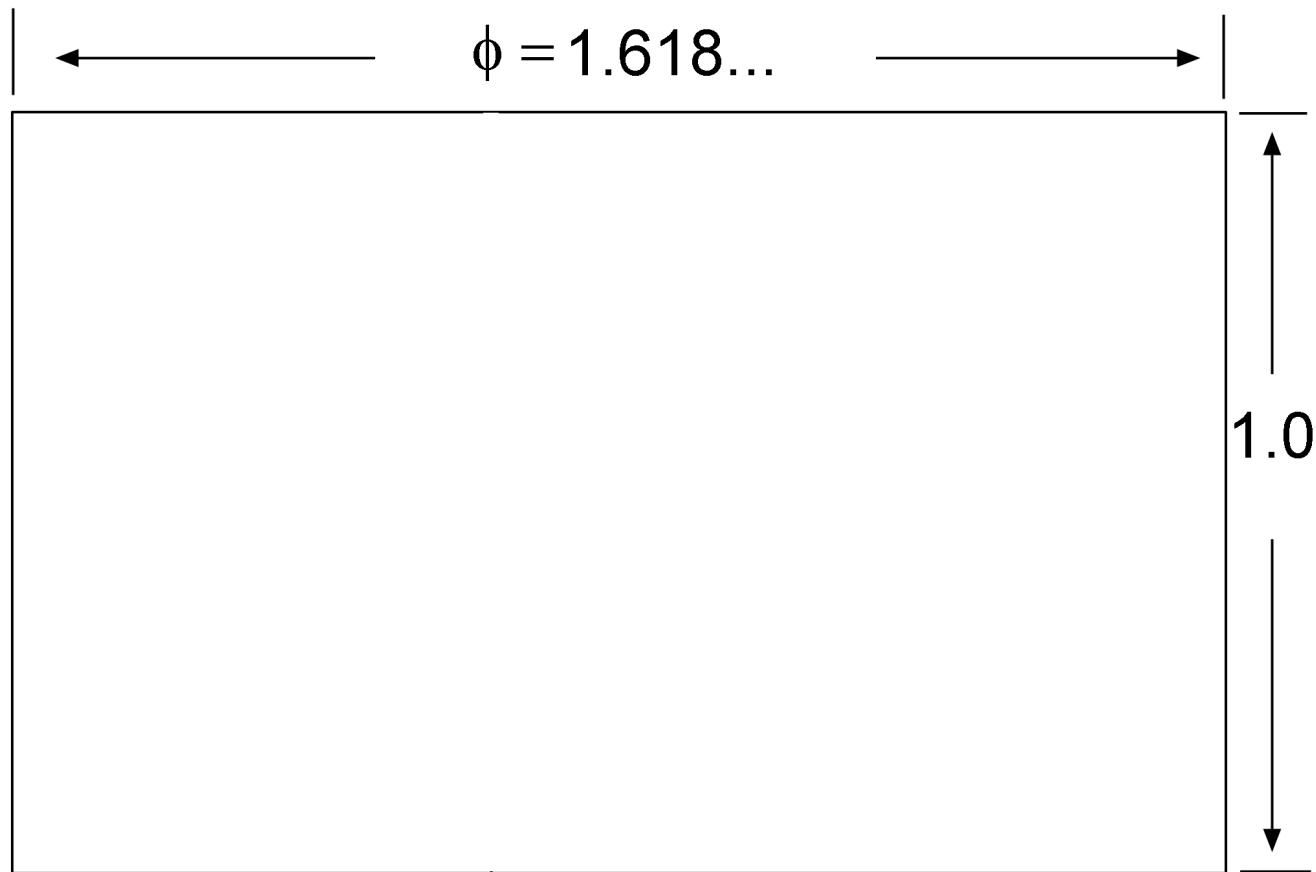
34 and 55 spirals



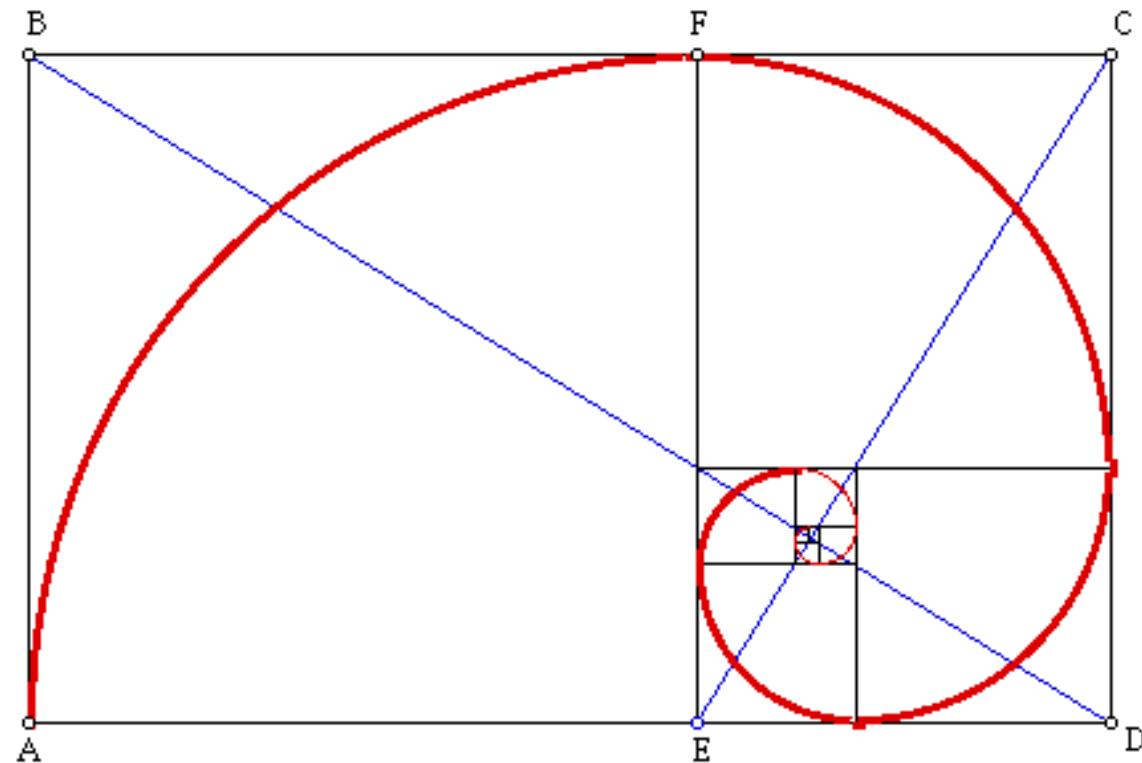
The Divine Proportion, $\Phi = 1.618\dots$

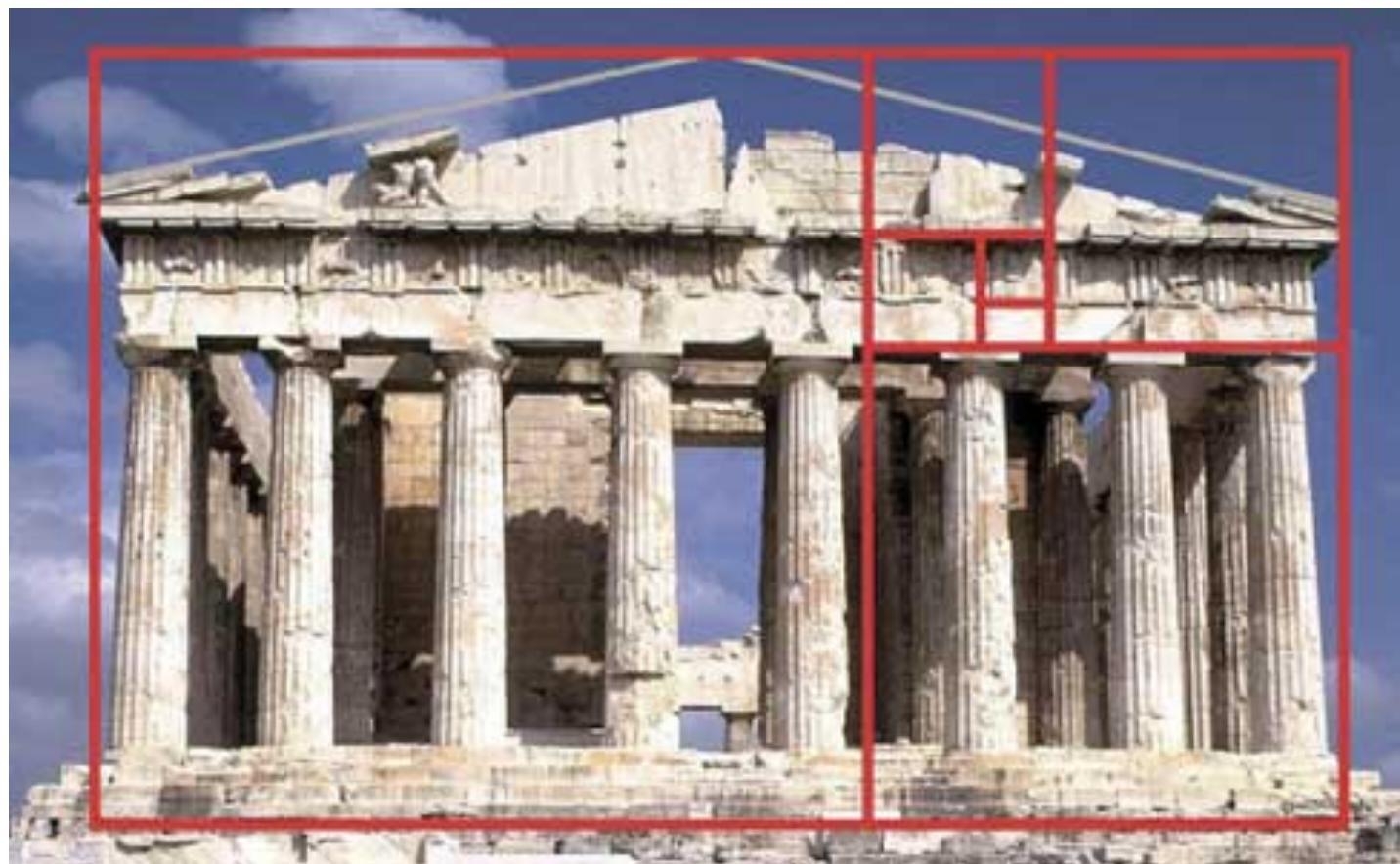


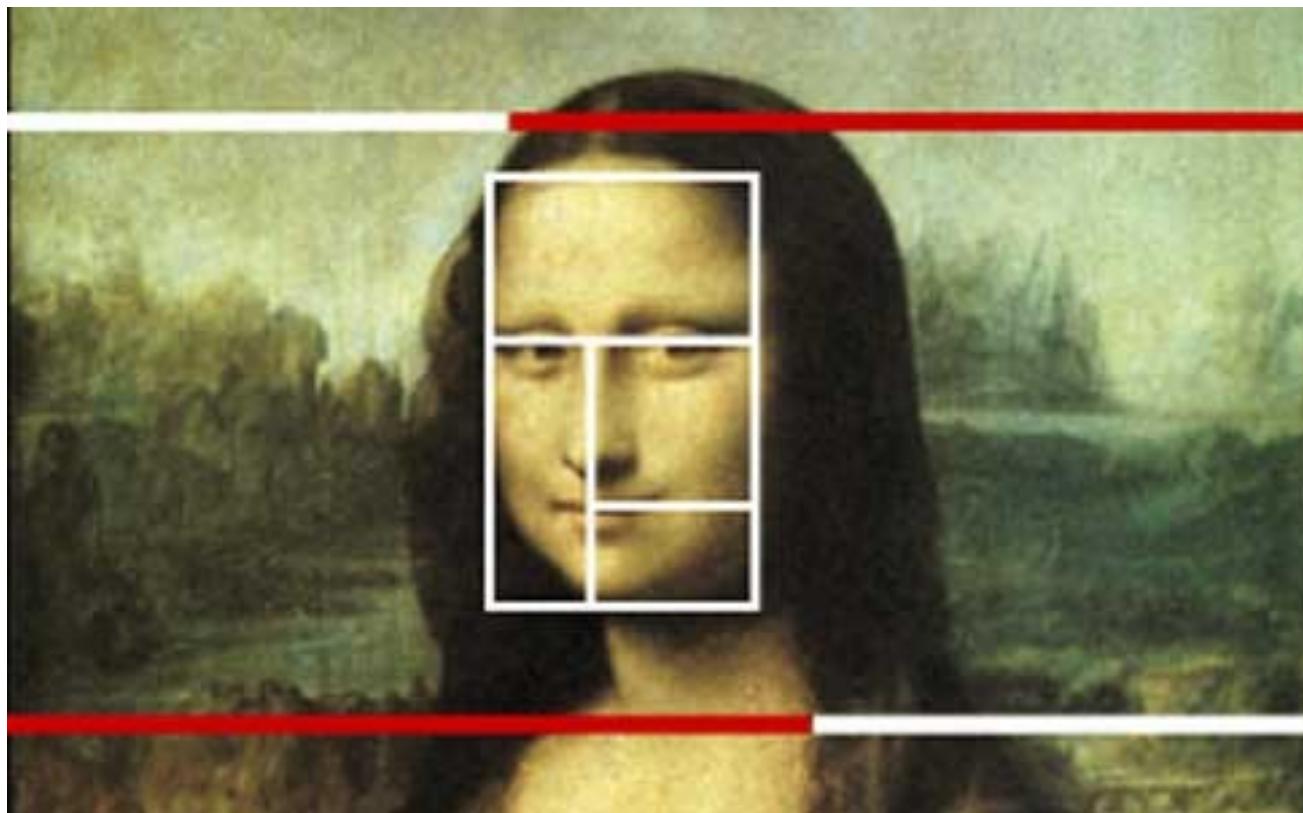
The Golden Rectangle



The Golden Rectangle





Φ 

A Divine Proportion



And now we generalize...

The Three-Dimensional Fibonacci Series

$$f_n = 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, \dots$$

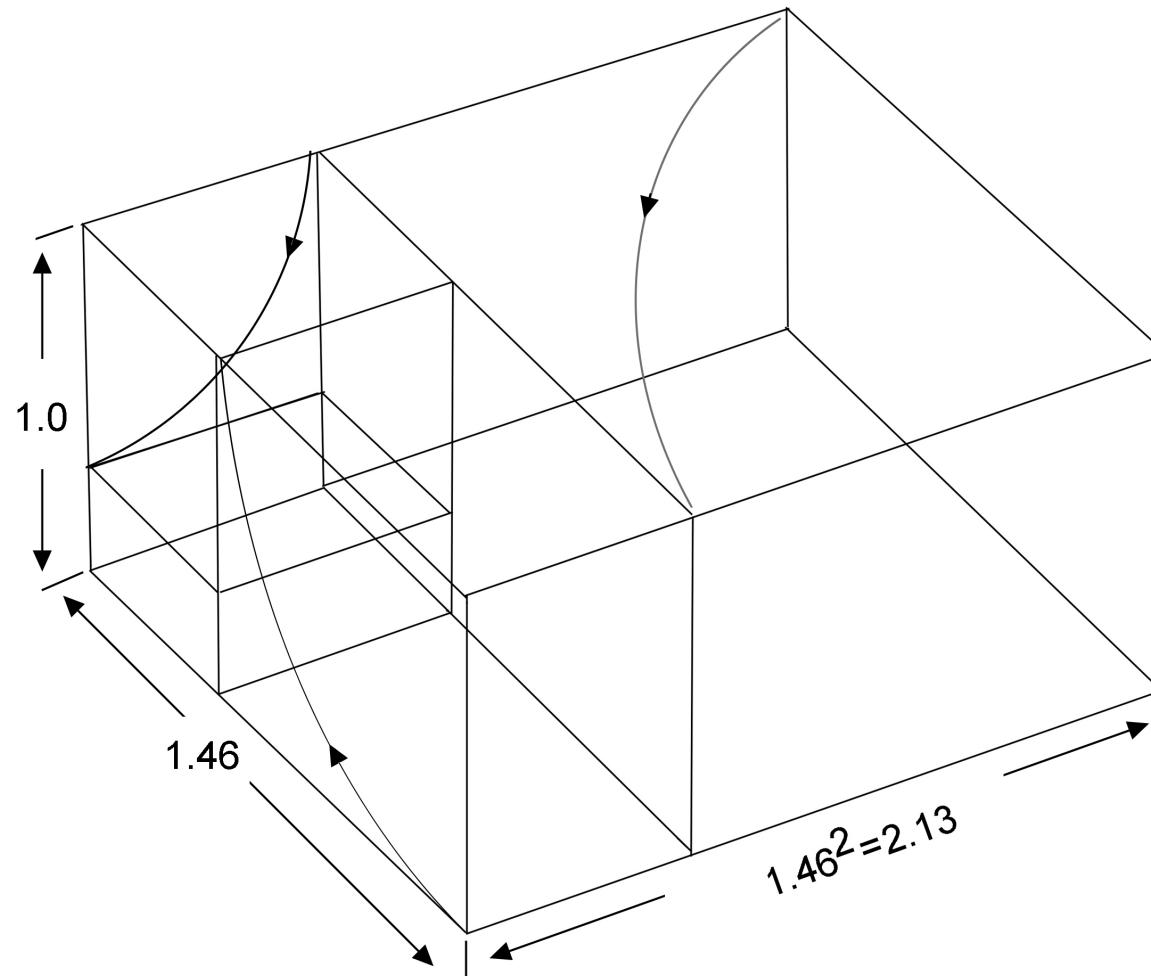
Rule: start with three ones
add second nearest neighbors

The Three-Dimensional Divine Proportion

$$\phi_3 = \lim_{n \rightarrow \infty} \frac{f_{n+1}}{f_n}$$
$$= 1.465 \dots$$

Then $D_f = \log(2)/\log(\Phi_3) = 1.815\dots$, Correct!

The Golden Brick



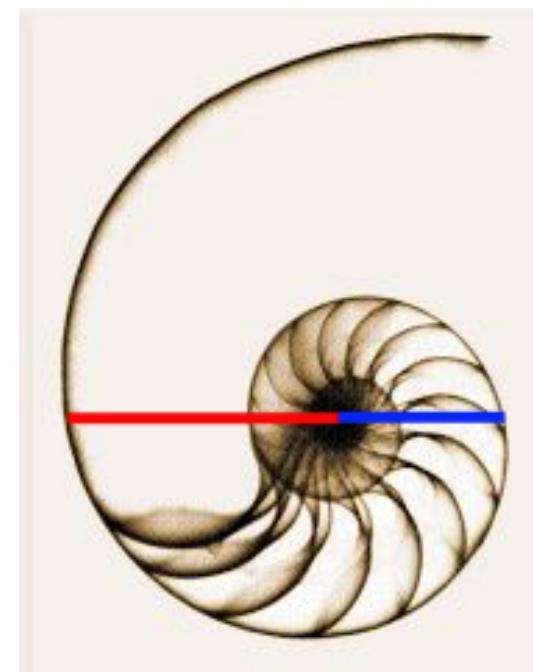
Fractal Aggregates

Spatial dimension , d	Fractal Dimension, D		
	Experiment	Simulations	Divine Proportion Theory
2	1.45 ± 0.05	1.44 ± 0.05	1.44
3	1.80 ± 0.10	1.80 ± 0.08	1.81
4	-	2.07 ± 0.15	2.15
5	-	2.33 ± 0.18	2.46
6	-	2.65 ± 0.25	2.76

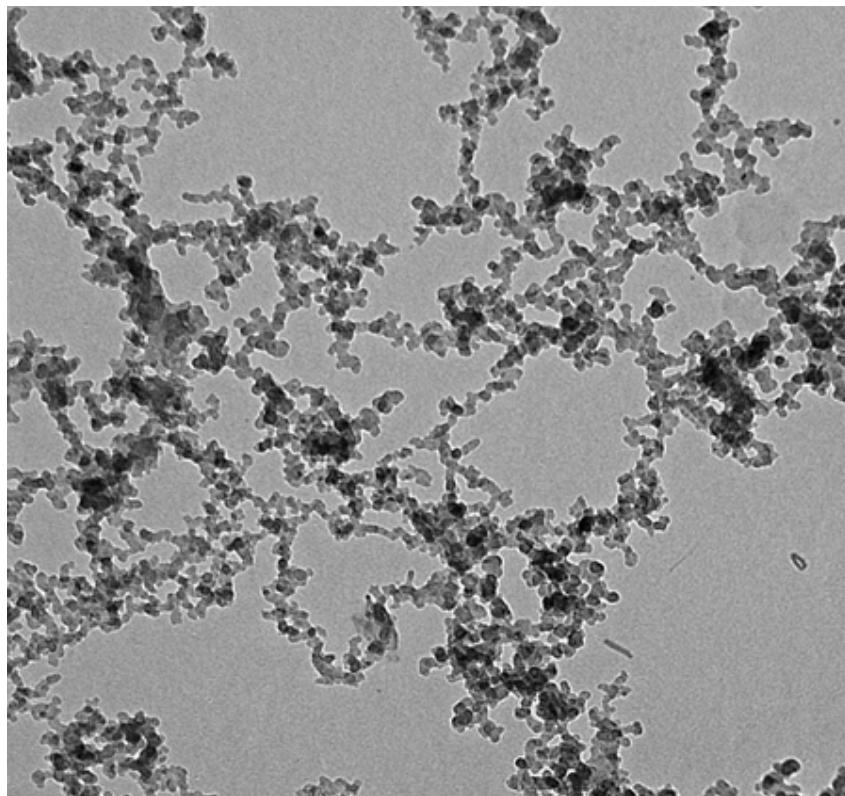
“...it shall widen and lengthen in the same unvarying proportions: and this simplest of laws is that which Nature tends to follow. The shell...grows in size but does not change in shape...”

D'Arcy Thompson

On Growth and Form



Soot and sunflowers



34000 30 min 2.tif
Propane 30 min
Print Mag: 43600x @ 3. in
14:44 07/29/04

100 nm
HV=100kV
Direct Mag: 34000x
AMT Camera System



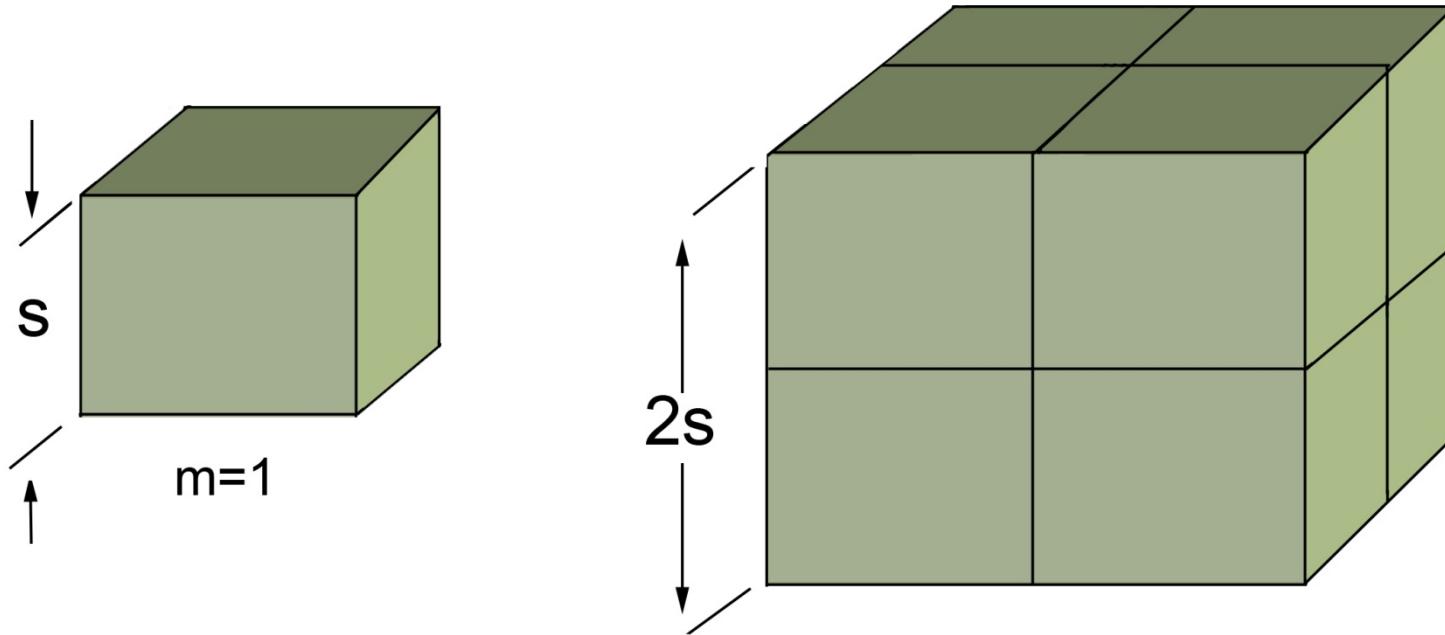
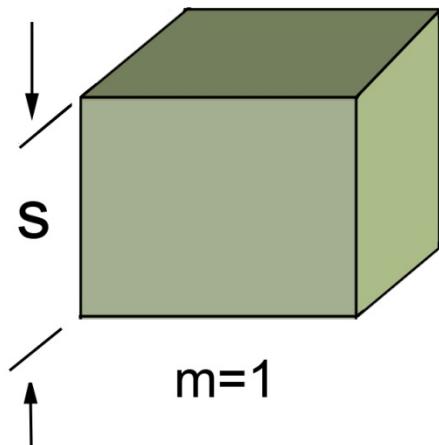
Thank you!



My students: M. Berg, S. Burchardt, J. Cai, R. Dhaudhabel, I. Elminyawi, G. Feke, D. Fry, S. Gangopadhyay, C. Gerving, S. E. Gilbertson, W. Hageman, W. Heinson, H. Huang, J. Hubbard, W. Kim, N. Lu, A. Mohammed, T. Mokhtari, A. Nepal, C. Oh, B.J. Olivier, F. Pierce, E. Ramer, S. Ren, G. Roberts, T. Rush, T. Taylor, G. M. Wang, and H.X. Zhang.
My colleague: A. Chakrabarti

Appendix

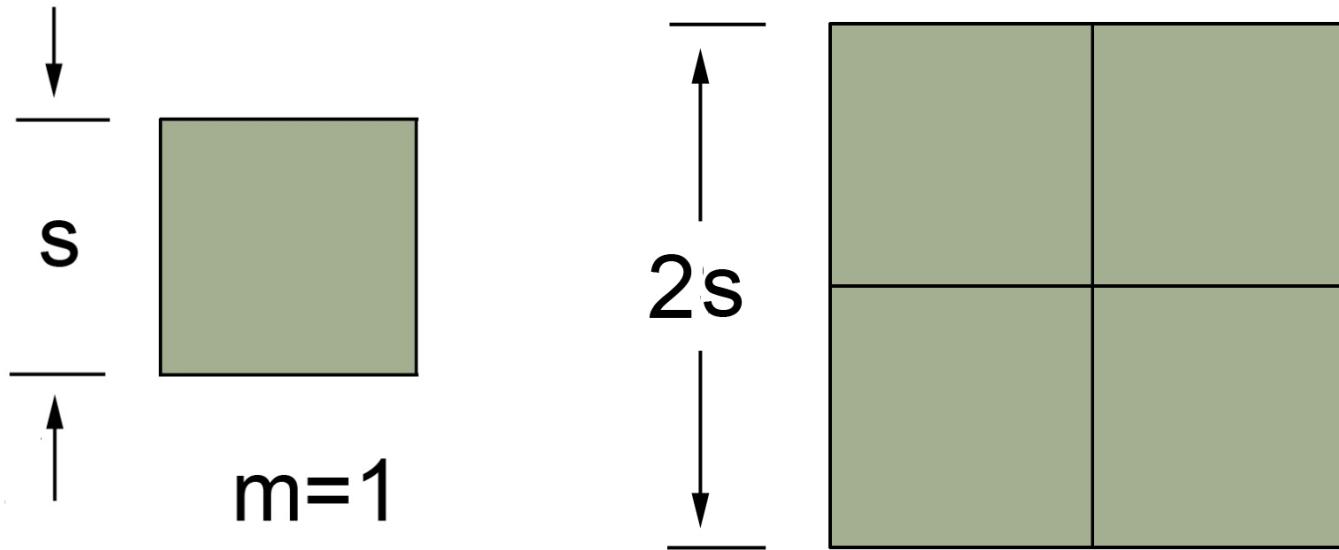
Scaling of the Cube



$m=8$

$$m \sim s^3$$

Scaling of the Plane



$$m \sim s^2$$

$1 \times 1 \times 1$



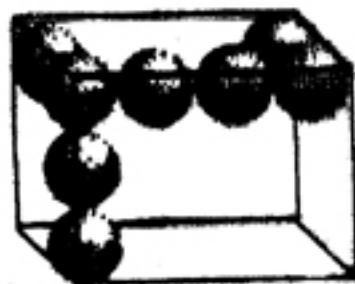
$1 \times 1 \times 2$



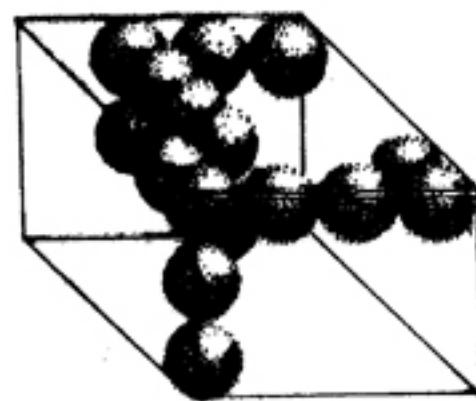
$1 \times 2 \times 3$



$2 \times 3 \times 4$



$3 \times 4 \times 6$



The d-Dimensional Fibonacci Series

a) Start with d “ones”

b) For integer $n \geq d$

$$f_{n+1,d} = f_{n,d} + f_{n-d+1,d}$$

Examples:

$$d=1; \quad f_{n,1} = 1, 2, 4, 8, 16, \dots \text{(geometric series)}$$

$$d=2; \quad f_{n,2} = 1, 1, 2, 3, 5, 8, 13, 21, 34, \dots \text{(Fibonacci series)}$$

$$d=3; \quad f_{n,3} = 1, 1, 1, 2, 3, 4, 6, 9, 13, 19$$

$$d=4; \quad f_{n,4} = 1, 1, 1, 1, 2, 3, 4, 5, 7, 10, 14, 19$$

The d-Dimensional Divine Proportion

$$\phi_d = \lim_{n \rightarrow \infty} \frac{f_{n+1,d}}{f_{n,d}}$$

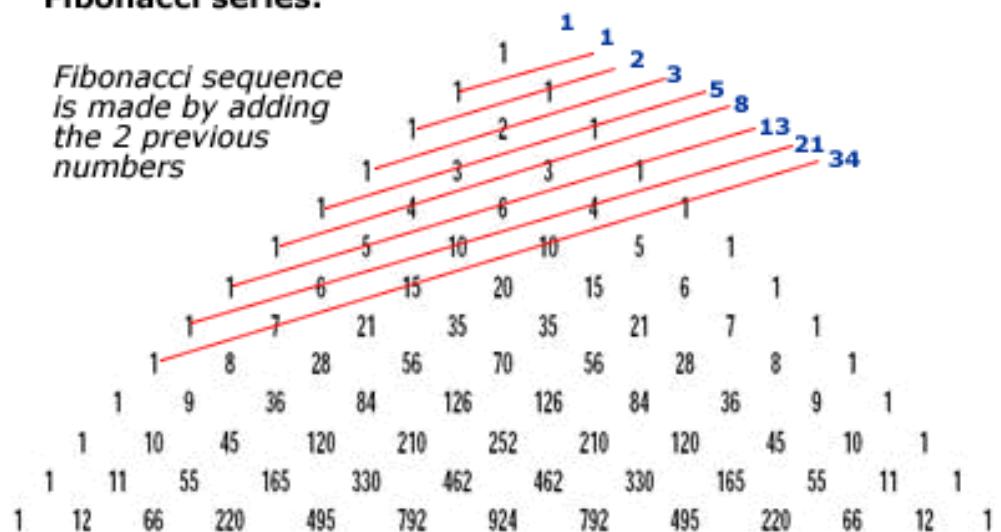
$$\phi_d^d - \phi_d^{d-1} - 1 = 0$$

Pascal's Triangle and the Fibonacci Numbers

		1				
	1	1				
	1	2	1			
	1	3	3	1		
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

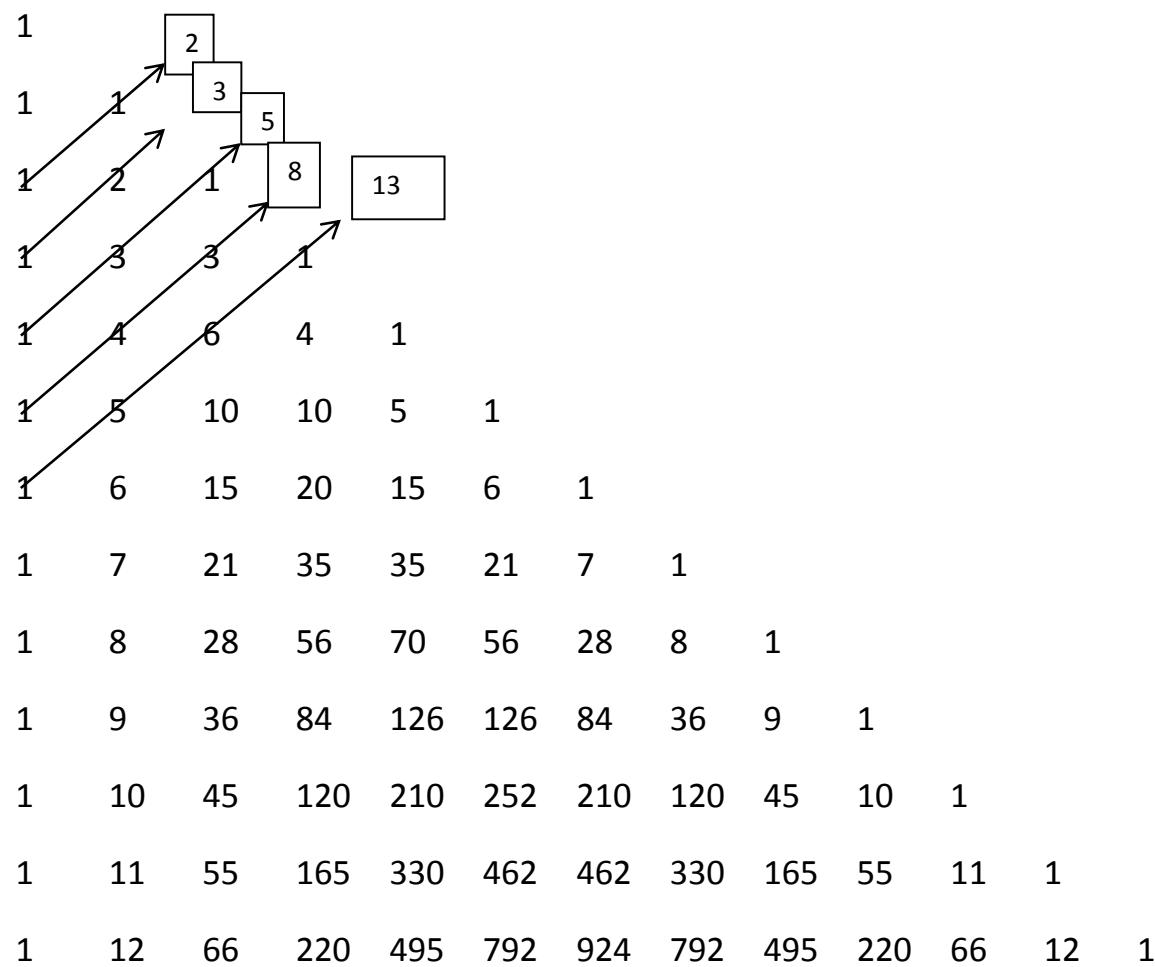
Fibonacci Sequence:
1 1 2 3 5 8 13 21 34
numbers on diagonals of the triangle add to the Fibonacci series.

*Fibonacci sequence
is made by adding
the 2 previous
numbers*



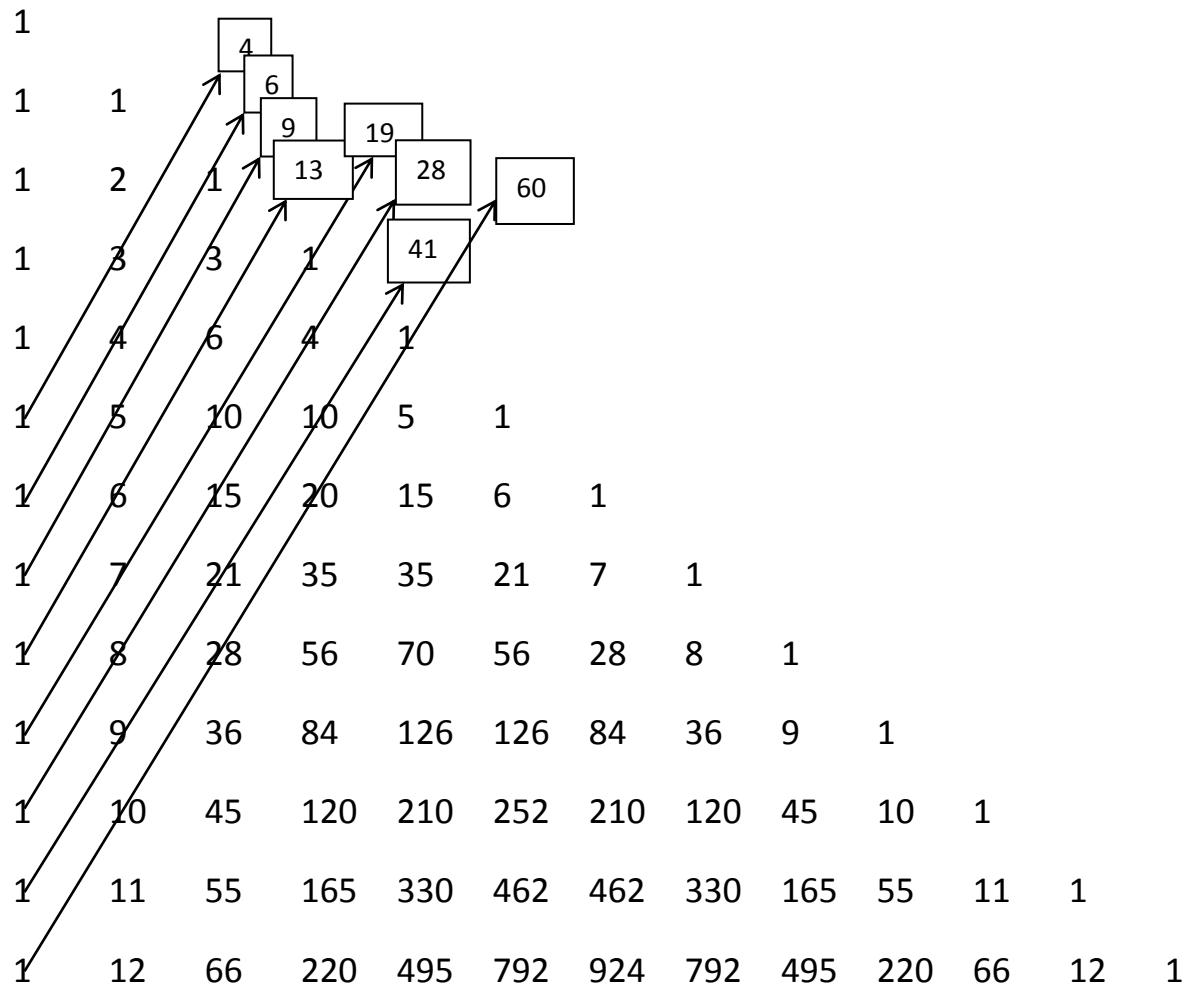
Fibonacci numbers (2d) in the Pascal Triangle

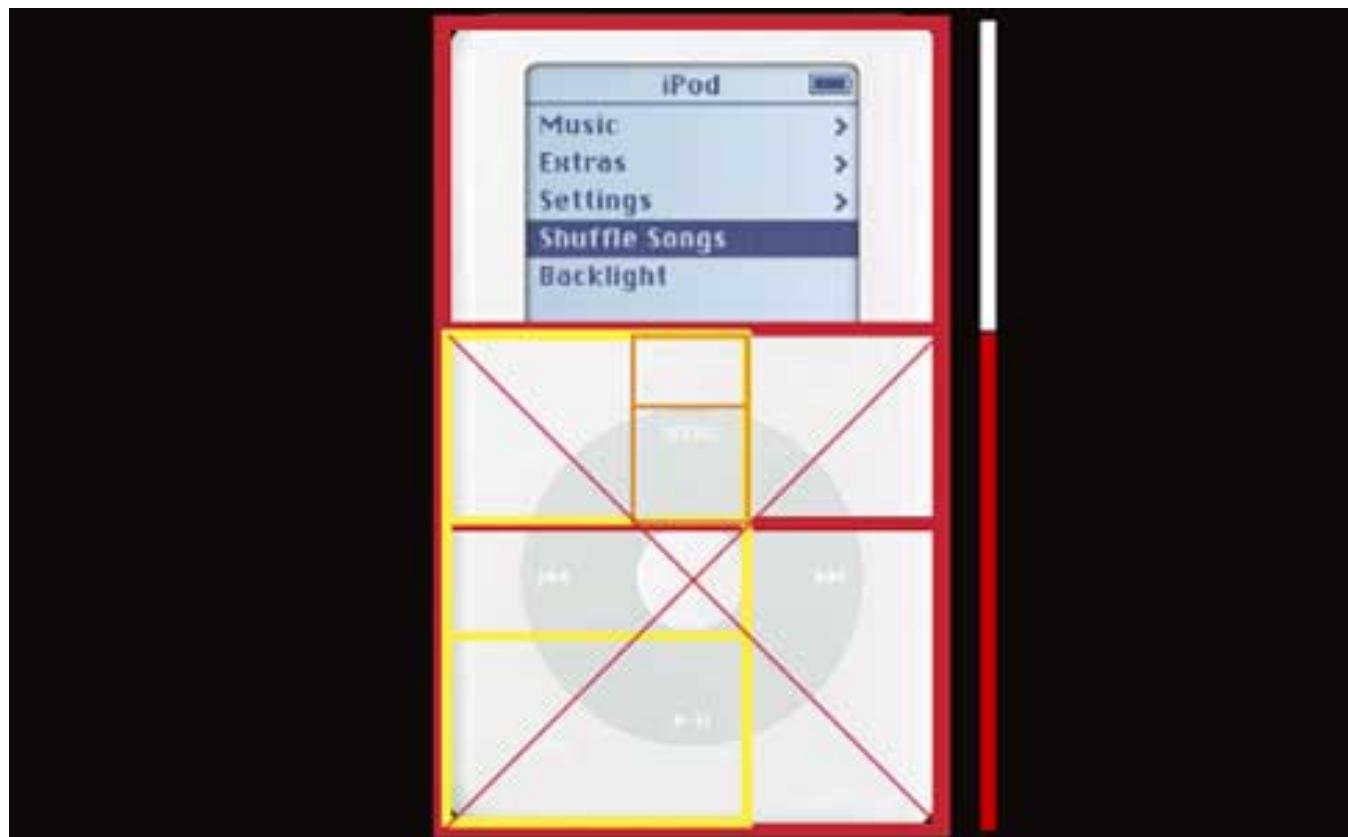
Pascal's Triangle



3d Fibonacci numbers in the Pascal Triangle

Pascal's Triangle





The Three-Dimensional Fibonacci Series

- $f_n = 1, 1, 1, 2, 3, 4, 6, 9, 13, 19, 28, \dots$
- Rule: start with three ones
Add second nearest neighbors

The Three-Dimensional Divine Proportion

$$\phi_3 = \lim_{n \rightarrow \infty} \frac{f_{n+1}}{f_n}$$
$$= 1.465 \dots$$

Scaling

$$m \sim s^d$$

Where d is the dimensionality of the object

$$\log m \sim \log s^d$$

$$\log m \sim d \log s$$