

A photograph of a light-colored dog sitting on a sandy beach, facing the ocean. The sun is setting behind a line of clouds, casting a warm orange glow over the water and the dog's back. The sky is a mix of blue and orange. The ocean waves are visible in the background.

A Universe of Light
Chris Sorensen
Physics
Kansas State University

Light Scattering

It's how we see the world. Every non-luminous thing we see, we see via light scattering.

Reflection is a special form of light scattering.

Non Invasive Probe of

Aerosol Systems

Colloid Science

Biophysics

Condensed Matter Physics

Aerosol Science

Atmospheric Visibility

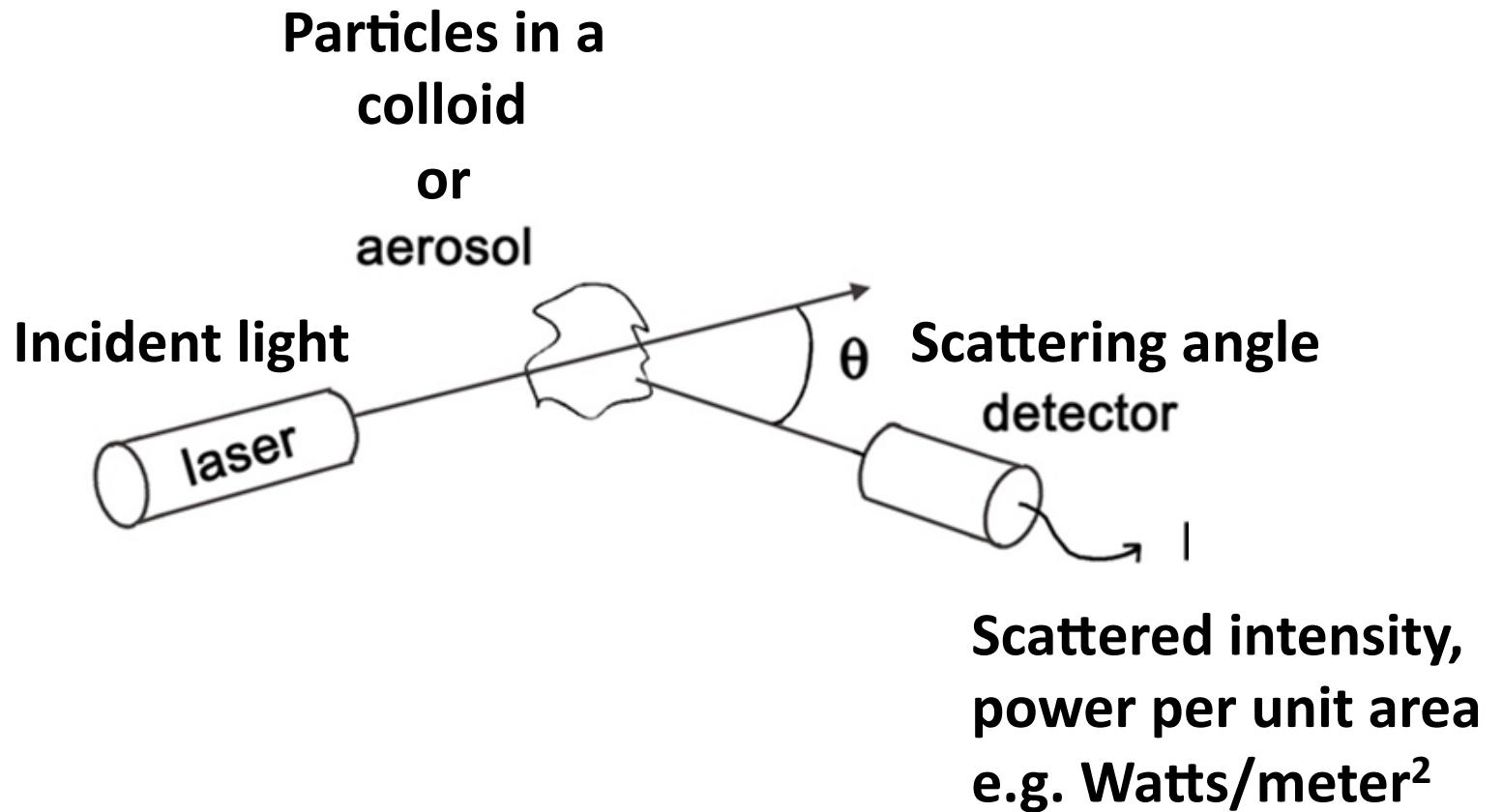
Earth's Radiation Budget

Problems Becoming More Complex

And its just neat stuff!

Scattering 101

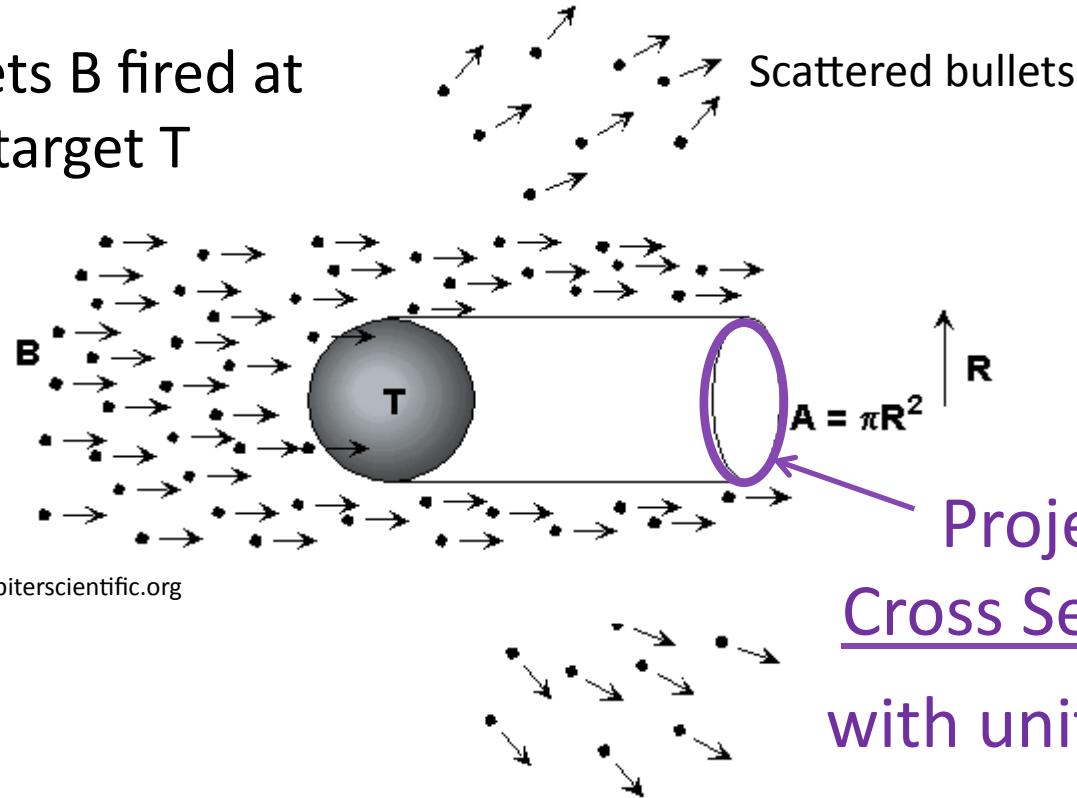
Configuration and Terms



Scattering 101

Cross Section

Bullets B fired at target T



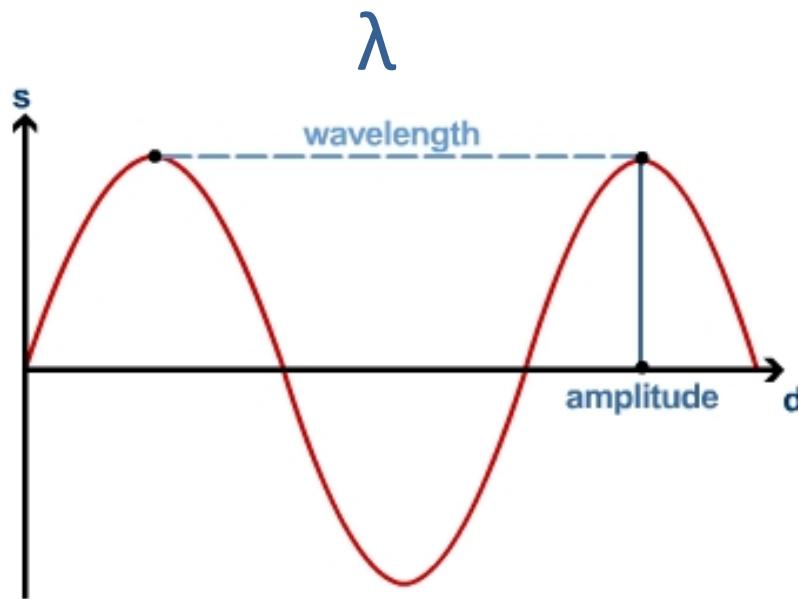
jupiterscientific.org

Projected area is the
Cross Section for scattering
with units of area \sim length².

Scattered intensity is proportional to scattering cross section and the number of things scattering, N

$$I \sim NC_{\text{sca}}$$

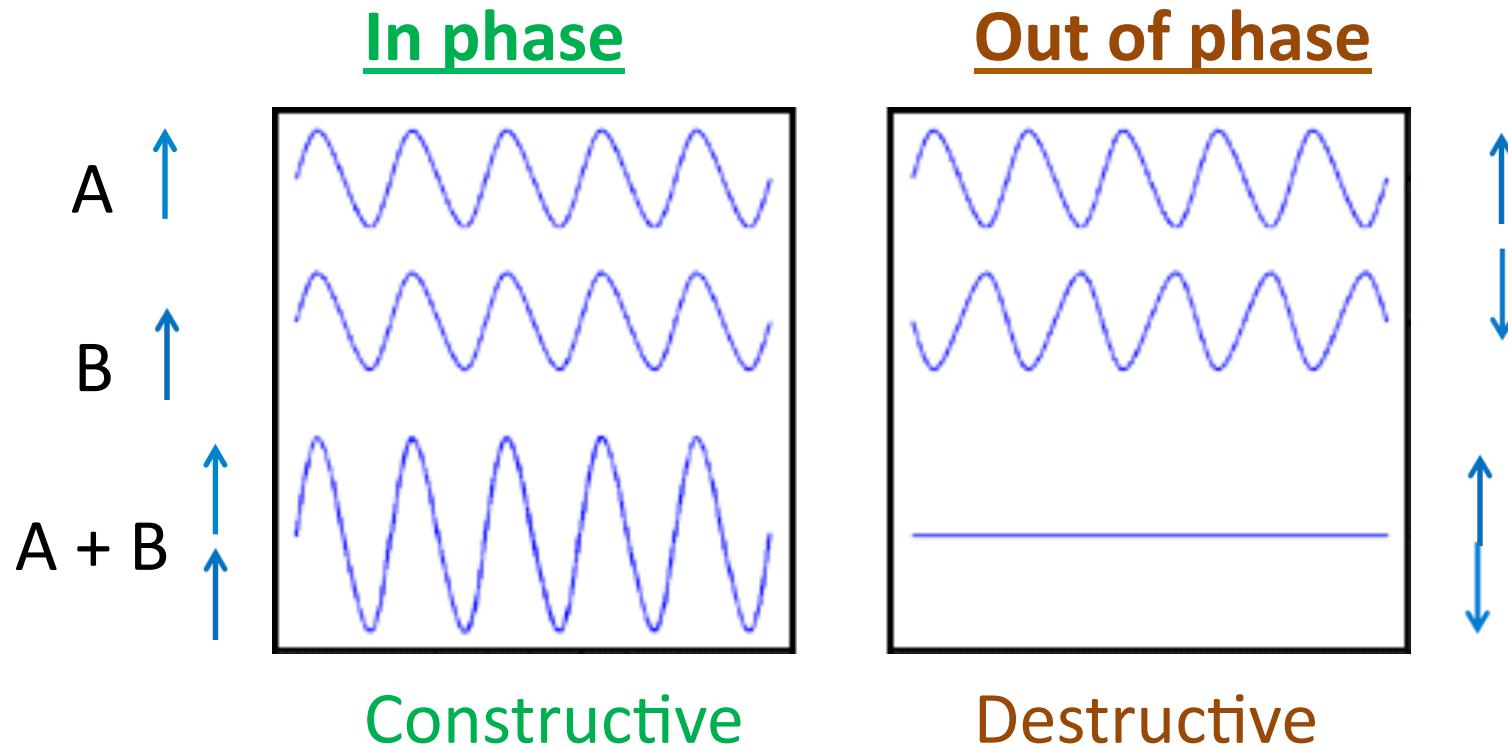
Waves 101



$$\text{Intensity (power/area)} \sim \text{Amplitude}^2$$

Waves 101

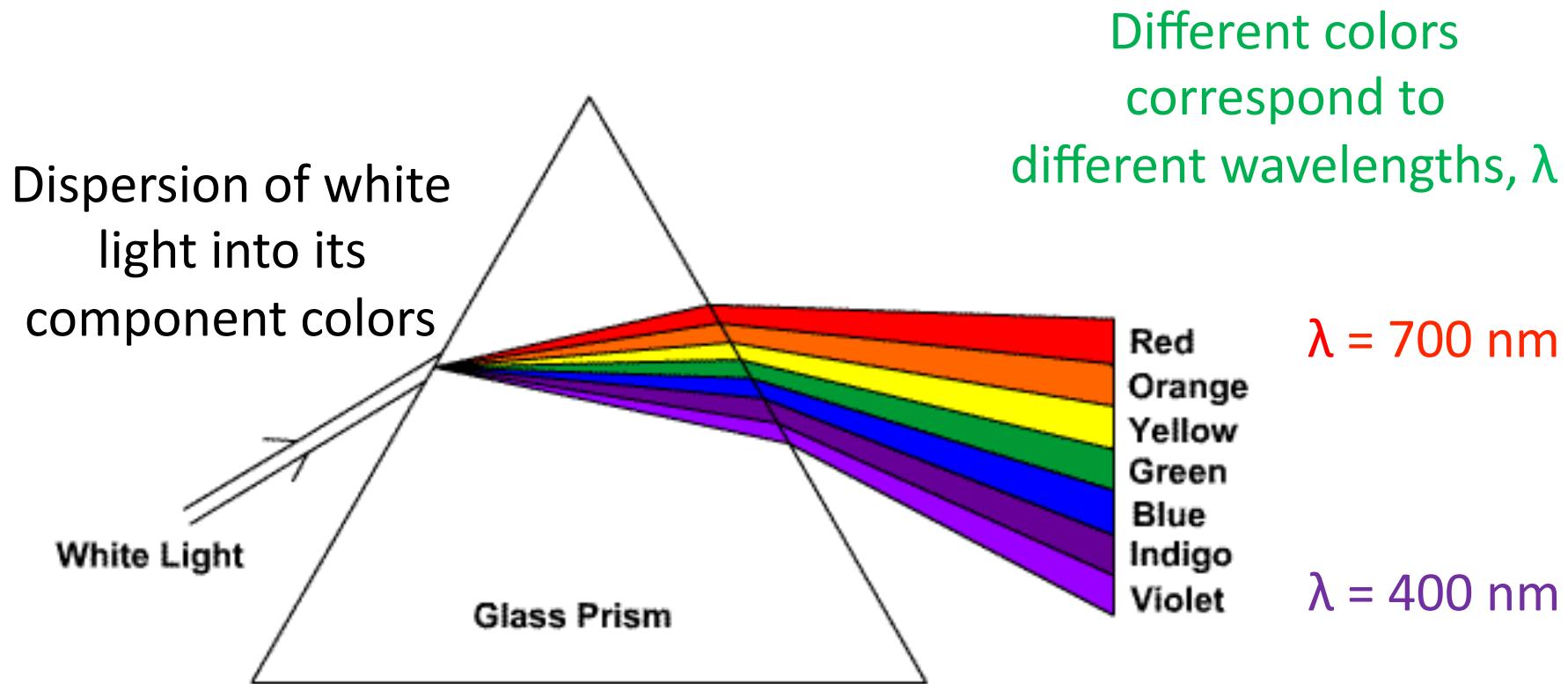
How Waves Combine. Wave Interference



Amplitude: doubles ($2x$)
Intensity: quadruples ($2^2 = 4$)

zero
zero

Optics 101



Bending of light at an interface is called refraction.

The refractive index of the material, **m**, determines the amount of bending.
Dispersion occurs because the refractive index depends on the wavelength.

Electromagnetism 101

Maxwell's Equations

And God Said

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho_v$$

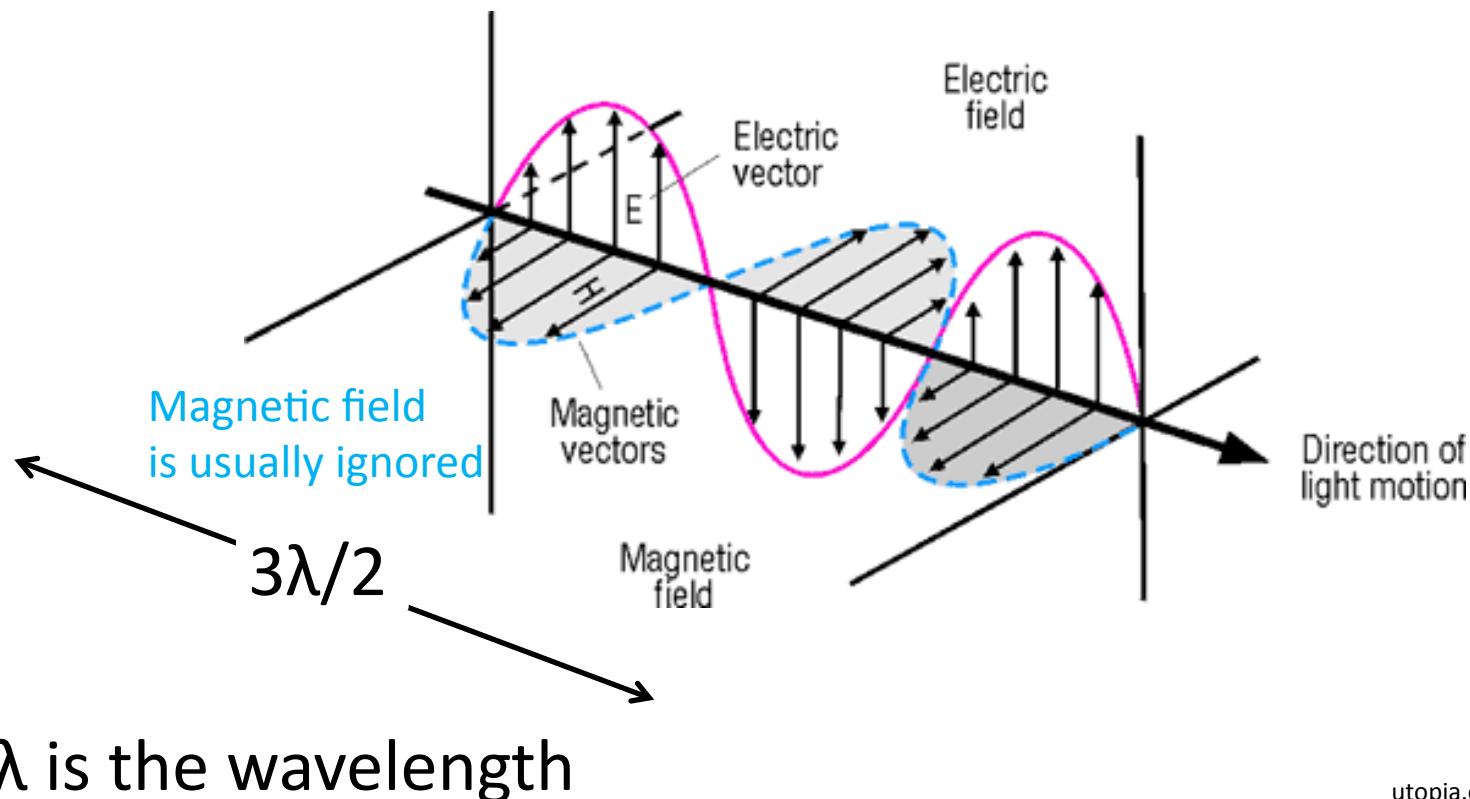
$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

and then there was light.

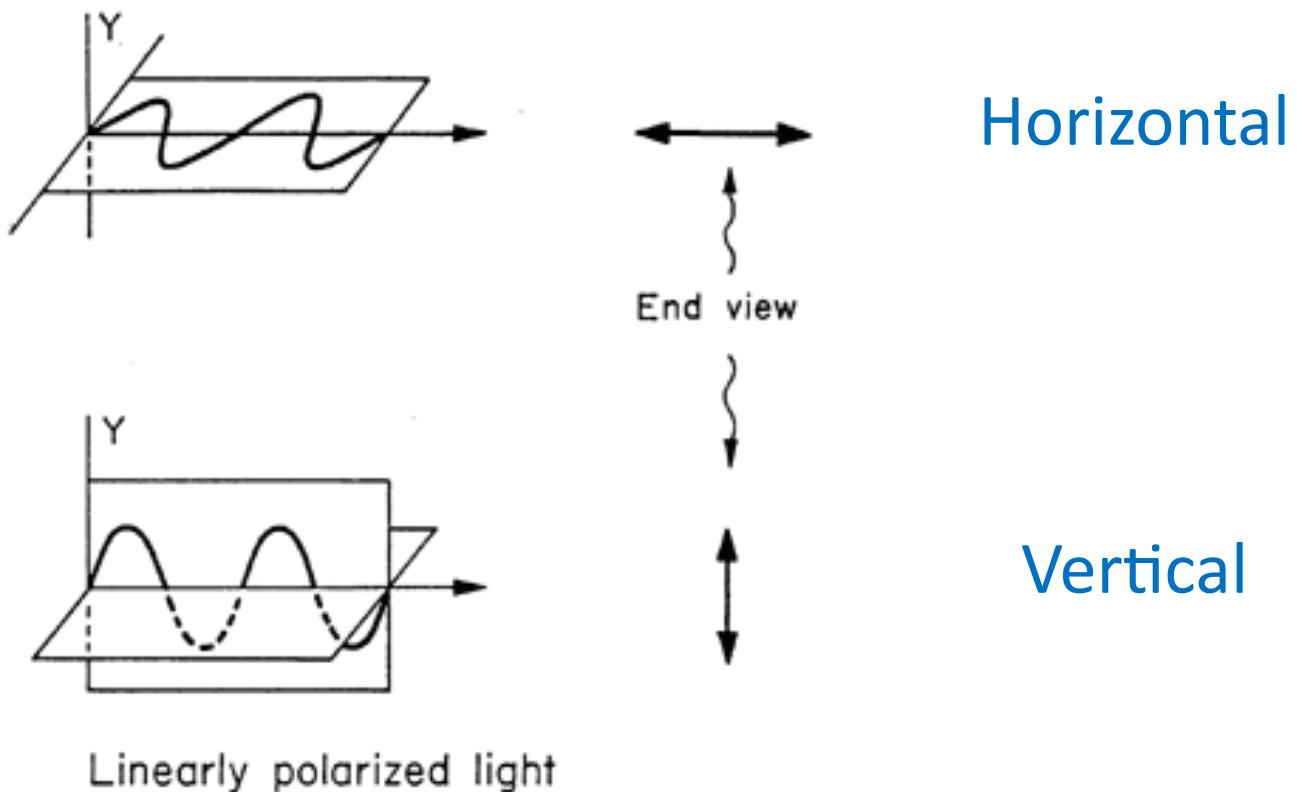
Light is an Electromagnetic Wave (while propagating)

Direction of the
electric field is the
Polarization direction



Polarization

The direction of the wiggle



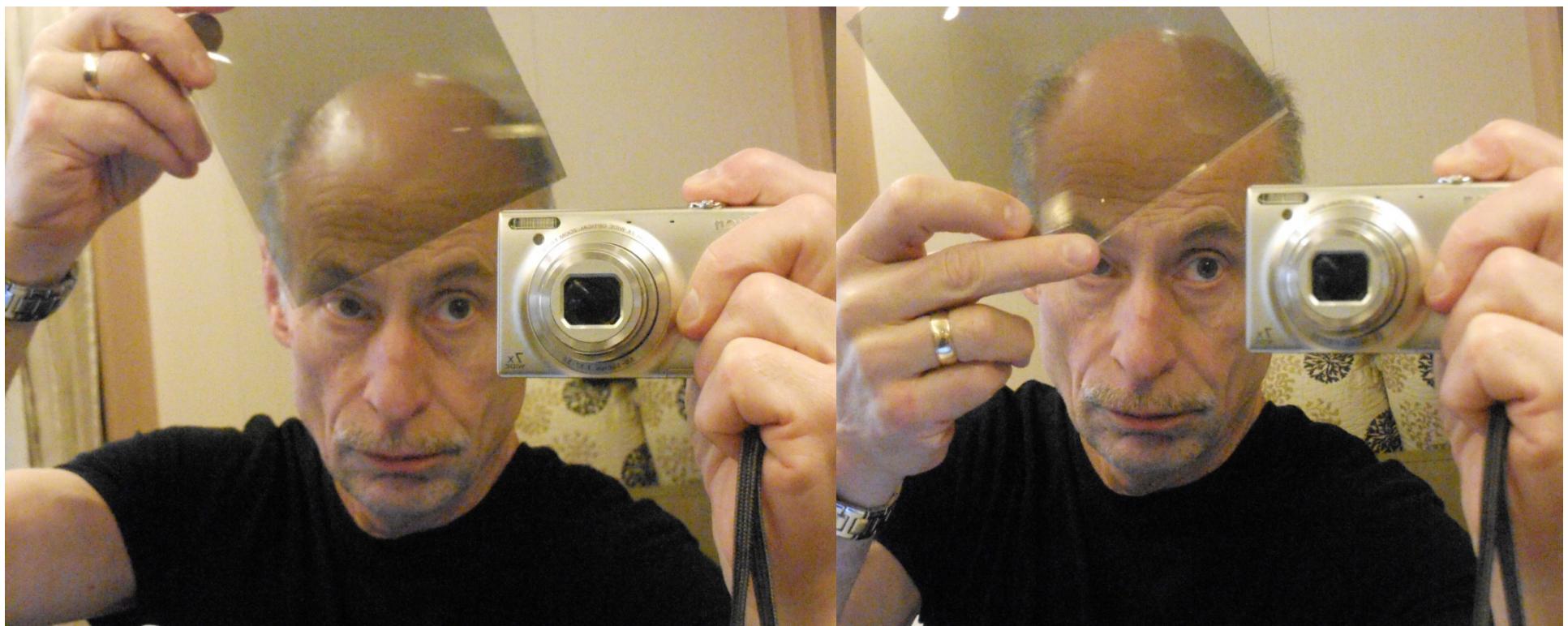
Polarization upon Reflection

Reduction of glare with use of polarizing filters

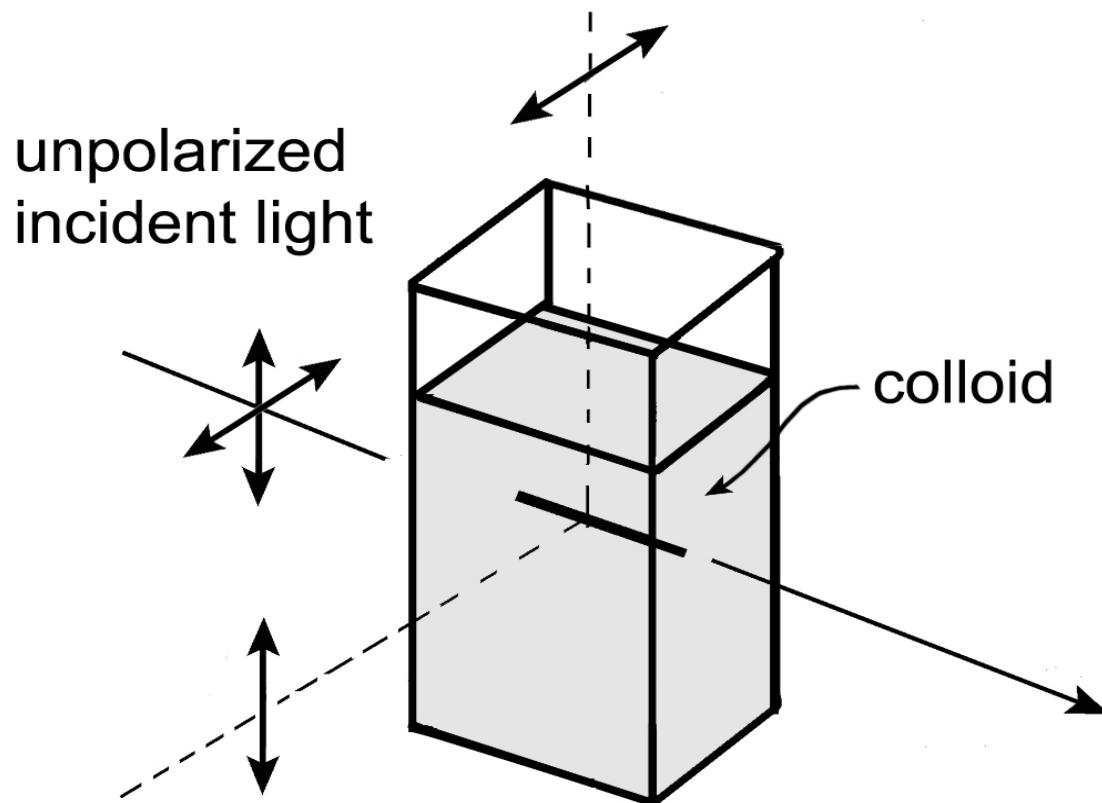


Polarization upon Reflection

SS Veendam



Polarization by scattering



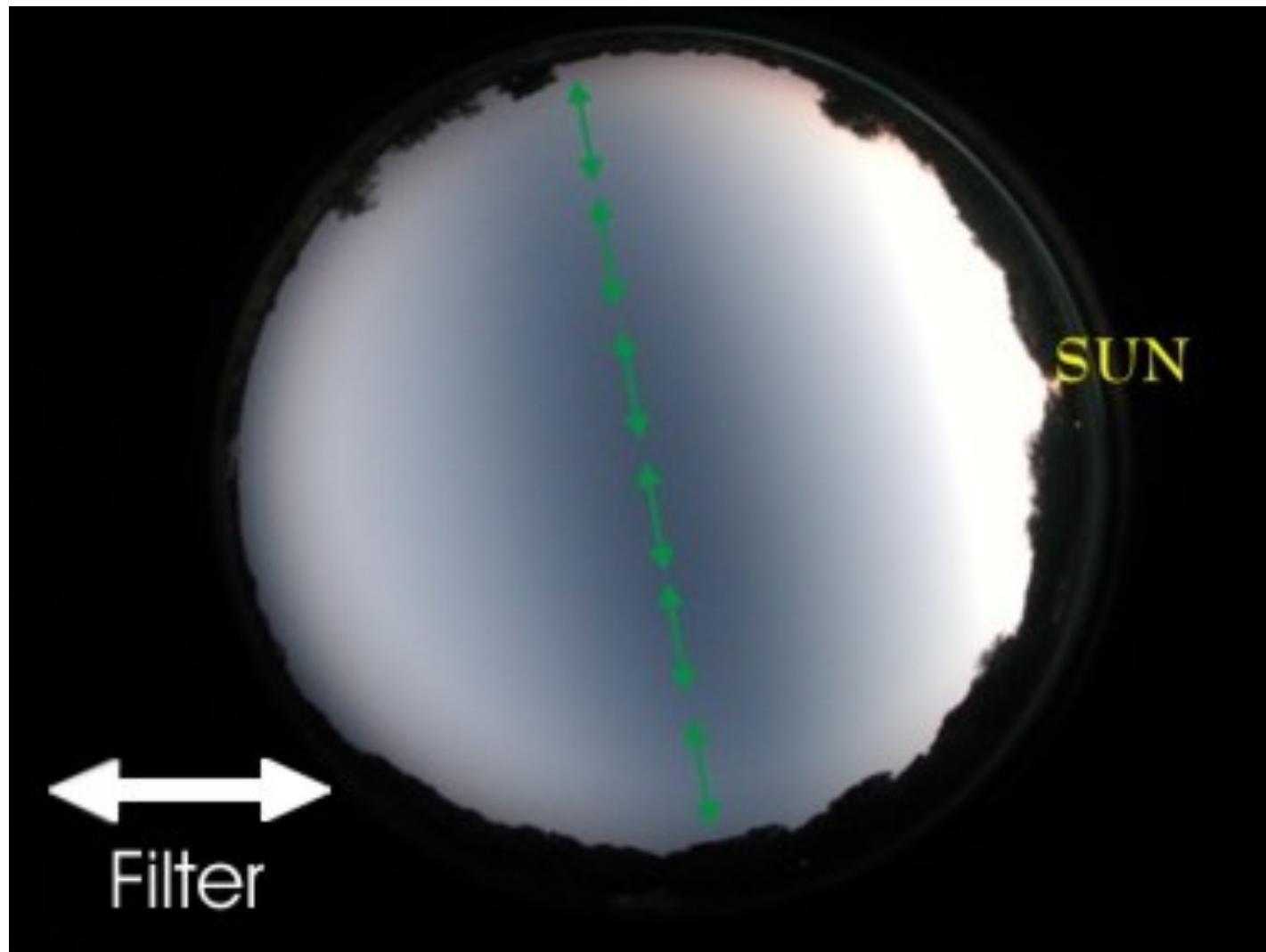


Polarization of the Entire Sky

Fisheye looking up to zenith

180° view

Maximum polarization is 90° from sun



Scattering by Particles, e.g.

- Atoms and molecules
- water droplets in fog, mist, rain
- Dust, smoke ...
- Colloids, e.g. milk ...

Key parameters:

1. Size, such as radius, R
2. Wavelength of the light, ... λ
3. Particle's relative refractive index, m
4. Shape (morphology)

We want to know how the scattered intensity depends on R , λ , m and the scattering angle θ .

Successive Steps to Light Scattering

1. Rayleigh Scattering

$R \ll \lambda$, very small, any shape

2. Rayleigh-Debye-Gans (RDG) Scattering

Any R , any shape

$$\rho = 4\pi[R/\lambda](m - 1) < 1$$

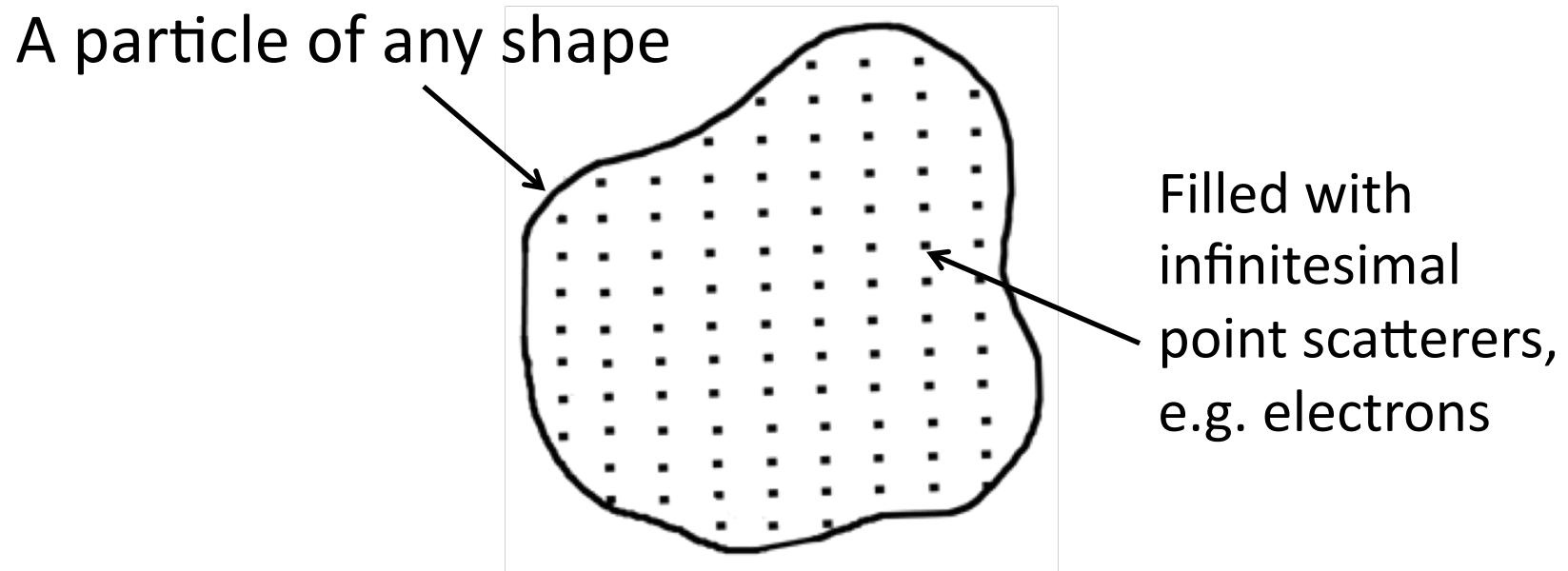
3. Mie Scattering

Any R , spheres

Any m

4. The Last Frontier: Any shape, any size, any m .

System of Scatterers



Point scatterers scatter the waves uniformly in all directions.

Scattering of Waves

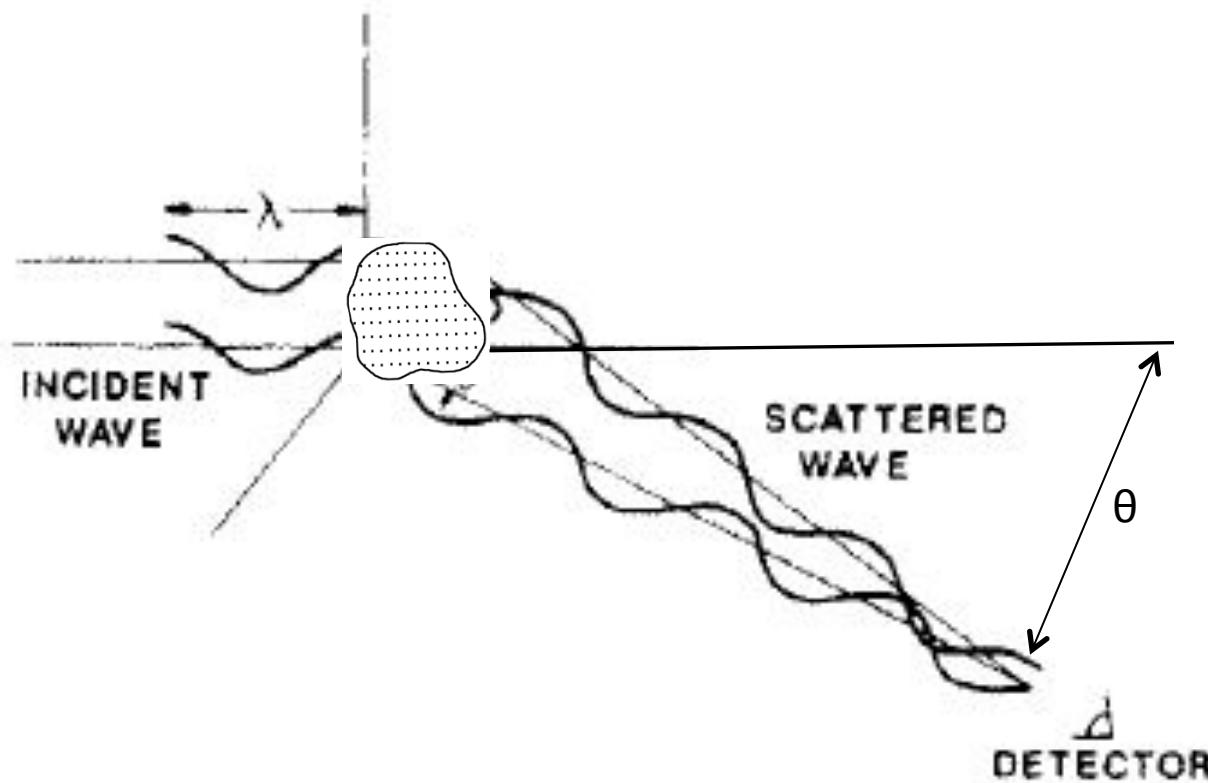


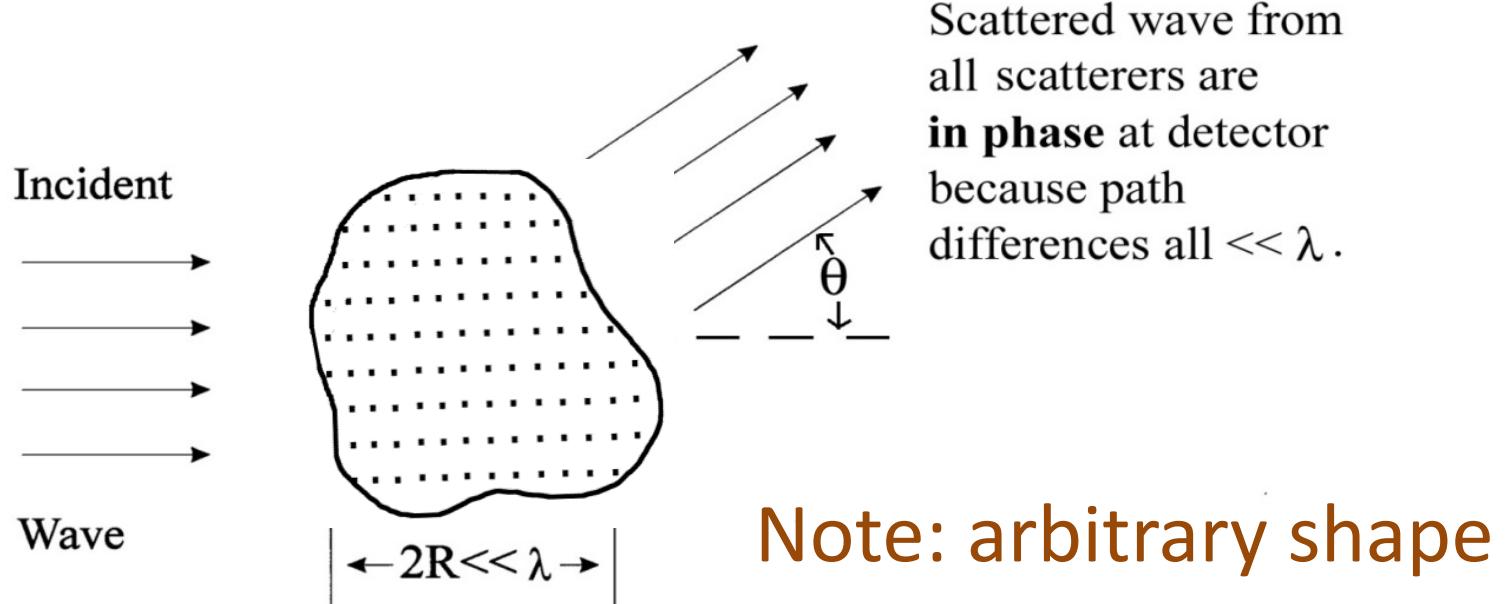
Figure 1. Schematic illustration of the interference between waves by different particles and how it determines the overall scattered field.

Rayleigh Scattering

Scattering from Small Particles (including molecules)

What do we mean by “small”?

---small compared to λ . **Size parameter** $2\pi R/\lambda < 1$.
(Only two length scales, R and λ).



Rayleigh Scattering (2)

Since $R \ll \lambda$, the point scatterers within particle

- see the same incident phase
- are all essentially the same distance from the detector

Thus they scatter in phase to the detector

(regardless of scattering angle).

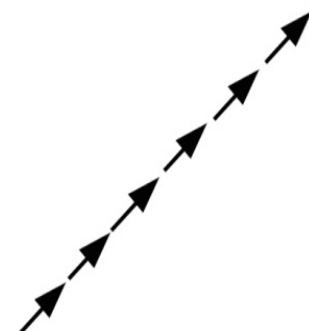
Total scattering amplitude, E_{sca}

$$E_{\text{sca}} \sim \text{Particle Volume} = V$$

Total scattered intensity

$$I_{\text{sca}} \sim E_{\text{sca}}^2 \sim V^2$$

$$\text{Hence } C_{\text{sca}} \sim V^2$$



Rayleigh Scattering (3)

Unit Analysis

Cross section units: area = (length)²

But so far we have

$$C_{\text{sca}} \sim V^2 = (\text{length})^6$$

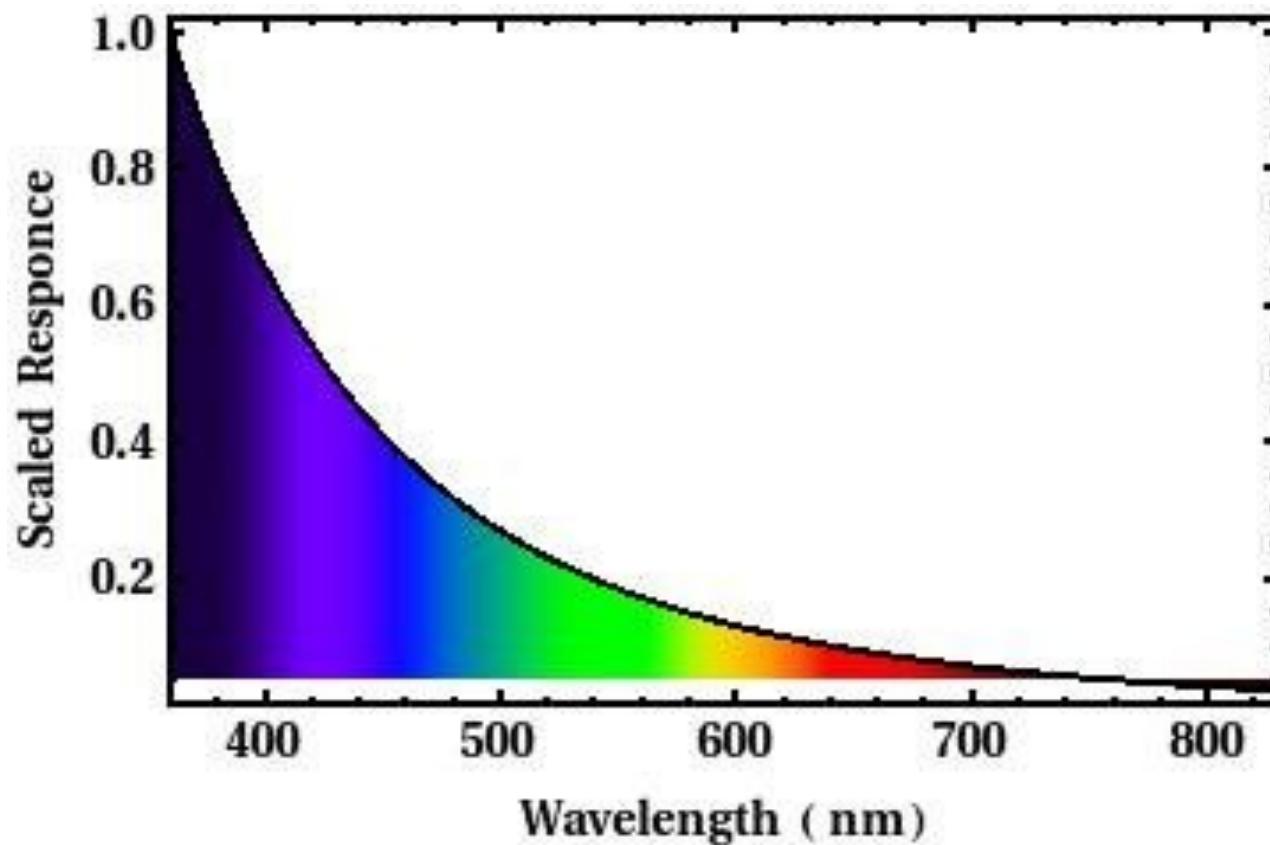
The only other length scale we have
is the wave length of the light, λ . So ...

We must have

$$C_{\text{sca}} \sim V^2/\lambda^4$$

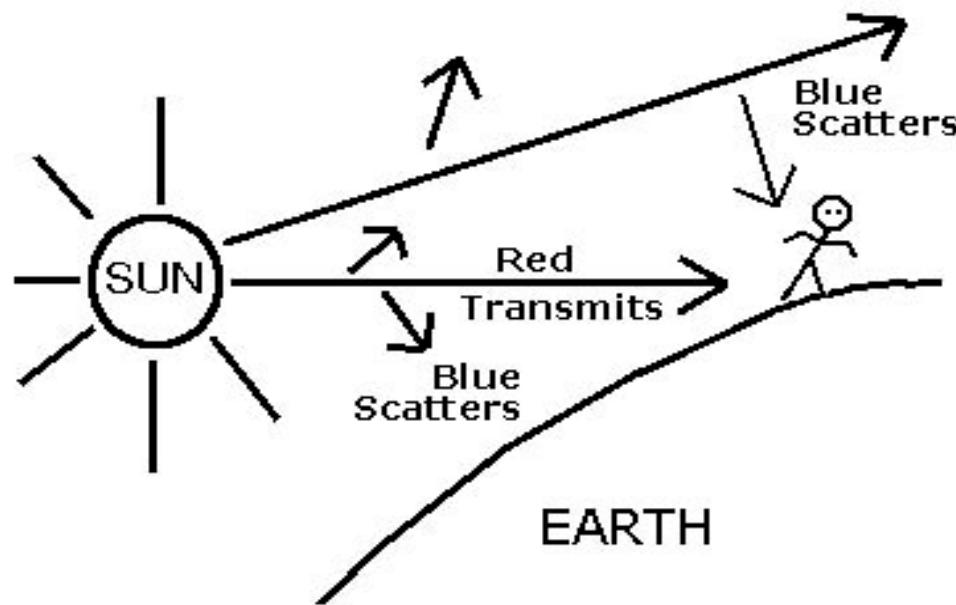
$$\sim R^6/\lambda^4 \text{ for spheres}$$

Rayleigh's $1/\lambda^4$ Law



Consequences of Rayleigh Scattering

1. Blue skies and red sunsets

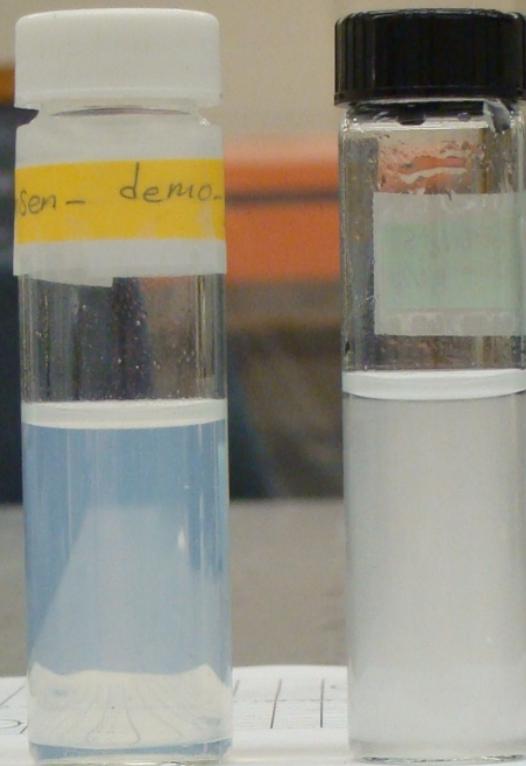


Rayleigh Scattering



Sunset over the Flint Hills of Kansas

Scattering



Transmission



Consequences of Rayleigh Scattering

2. Isotropic scattering

i.e. same in all directions, independent of θ

e.g. recall the uniformity of the sky.

Consequences of Rayleigh Scattering

3. The Tyndall Effect; increased scattering as a system coarsens, e.g., condensation of a fog.

$I_{\text{sca}} = \text{Number of particles} \times \text{cross section/part}$

$$= NC_{\text{sca}} \sim N V^2 / \lambda^4$$

$$\sim NV \cdot V / \lambda^4$$

total stuff (conserved hence constant)



Hence $I_{\text{sca}} \sim V_{\text{particle}}$

The Tyndall Effect



The Tyndall Effect

In Chacabuco Chile

SS Veendam

Feb 23, 2013



Scattering from Spheres

Any size, any refractive index

The Mie Equations

$$\sigma_{\text{sca}} = \left(\lambda^2/2\pi\right) \sum_{n=1}^{\infty} (2n+1) \left\{ |a_n|^2 + |b_n|^2 \right\}$$

$$\sigma_{\text{ext}} = \left(\lambda^2/2\pi\right) \sum_{n=1}^{\infty} (2n+1) \left\{ \text{Re}(a_n + b_n) \right\}$$

$$a_n = \frac{\psi_n(\alpha)\psi'_n(\beta) - m\psi_n(\beta)\psi'_n(\alpha)}{\zeta_n(\alpha)\psi'_n(\beta) - m\psi_n(\beta)\zeta'_n(\alpha)}$$

$$b_n = \frac{m\psi_n(\alpha)\psi'_n(\beta) - \psi_n(\beta)\psi'_n(\alpha)}{m\zeta_n(\alpha)\psi'_n(\beta) - \psi_n(\beta)\zeta'_n(\alpha)}$$

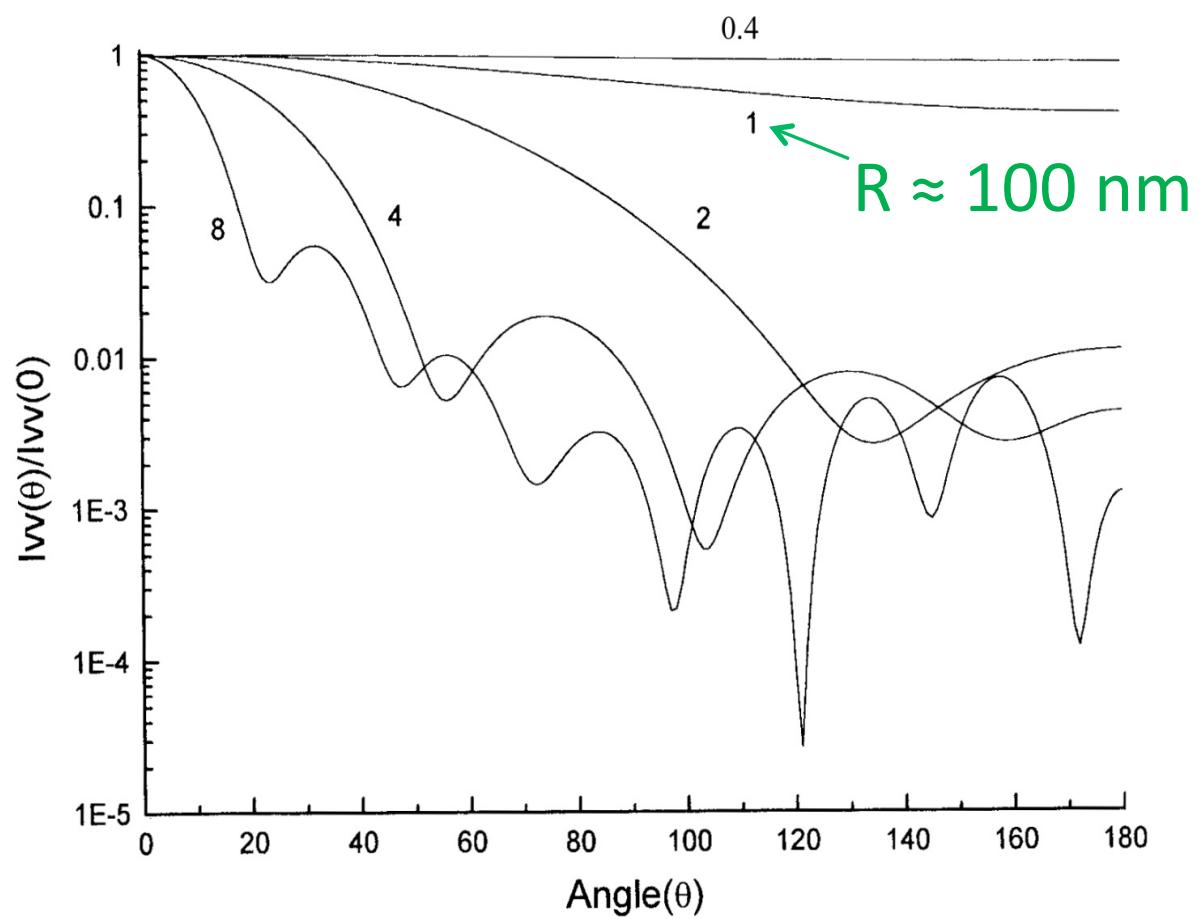
$$\alpha = k_2 a = 2\pi a / \lambda = 2\pi m_2 a / \lambda_o$$

$$\beta = k_1 a = 2\pi m_1 a / \lambda_o = ma$$

$$\begin{aligned}
& \sqrt{n' + m'} \alpha^{n' + 1} \beta^{m' + 1} \sqrt{n + m} \alpha^{n + 1} \beta^{m + 1} \\
I_{mm'm'n'}^{(1)}(q, l, \mu) &= m \left[\frac{n' \sqrt{(n+1)^2 - m'^2}}{2n'+1} G_{mm'm'n'+1}(q, l, \mu) - \frac{(n'+1) \sqrt{n^2 - m'^2}}{2n'+1} G_{mm'm'n'-1}(q, l, \mu) \right] + \\
& + m' \left[\frac{n \sqrt{(n+1)^2 - m^2}}{2n+1} G_{mn+1m'n'}(q, l, \mu) - \frac{(n+1) \sqrt{n^2 - m^2}}{2n+1} G_{mn-1m'n'}(q, l, \mu) \right], \quad (\text{A20}) \\
I_{mm'm'n'}^{(2)}(q, l, \mu) &= |mm'| G_{mm'm'n'}(q, l, \mu) + \frac{1}{(2n'+1)(2n+1)} \begin{bmatrix} (n'+1) \sqrt{n'^2 - m'^2} (n+1) \sqrt{n^2 - m^2} G_{mn-1m'n'-1}(q, l, \mu) + \\ n' \sqrt{(n'+1)^2 - m'^2} n \sqrt{(n+1)^2 - m^2} G_{mn+1m'n'+1}(q, l, \mu) - \\ (n'+1) \sqrt{n'^2 - m'^2} n \sqrt{(n+1)^2 - m^2} G_{mn+1m'n'-1}(q, l, \mu) - \\ n' \sqrt{(n'+1)^2 - m'^2} (n+1) \sqrt{n^2 - m^2} G_{mn-1m'n'+1}(q, l, \mu) \end{bmatrix}, \quad (\text{A21}) \\
L_{mm'm'n'}^{(00)}(k, l, k', l', \mu) &= m' Y_{mm'm'n'}(k, l, k', l', \mu), \quad (\text{A22}) \\
L_{mm'm'n'}^{(01)}(k, l, k', l', \mu) &= \frac{1}{2n'+1} \left[n' \sqrt{(n'+1)^2 - m'^2} Y_{mm'm'n'+1}(k, l, k', l', \mu) - (n'+1) \sqrt{n'^2 - m'^2} Y_{mm'm'n'-1}(k, l, k', l', \mu) \right], \quad (\text{A23}) \\
L_{mm'm'n'}^{(10)}(k, l, \mu) &= m' G_{mm'm'n'}(k, l, \mu), \quad (\text{A24}) \\
L_{mm'm'n'}^{(11)}(k, l, \mu) &= \frac{1}{2n'+1} \left[n' \sqrt{(n'+1)^2 - m'^2} G_{mm'm'n'+1}(k, l, \mu) - (n'+1) \sqrt{n'^2 - m'^2} G_{mm'm'n'-1}(k, l, \mu) \right], \quad (\text{A25}) \\
\Phi_m^{(0)}(k, l, k', l', \mu) &= \frac{1 + (-1)^{m+k\mu}}{2} \sum_{p=0}^{\mu} C_{|\mu|}^p (a_{lm})^p (b_{lm})^{\mu-p} [(a_{l'm} - ib_{l'm}) \Psi_{m+k}(k, p, \mu-p) + (a_{l'm} + ib_{l'm}) \Psi_{m-k}(k, p, \mu-p)], \quad (\text{A26}) \\
\Phi_m^{(1)}(k, l, k', l', \mu) &= \frac{k [1 + (-1)^{m+k\mu}]}{2} \sum_{p=0}^{\mu} C_{|\mu|}^p (a_{lm})^p (b_{lm})^{\mu-p} [(b_{l'm} - ia_{l'm}) \Psi_{m+k}(k, p, \mu-p) + (b_{l'm} + ia_{l'm}) \Psi_{m-k}(k, p, \mu-p)], \quad (\text{A27}) \\
\text{e } a_{l'm} \text{ and } b_{l'm} \text{ are coefficients in Eq. (3) and} \\
\Psi_{\tau}(k, p, t) &= \frac{i\pi e^{\frac{i\pi(\frac{k}{2}-p-1)}{2}}}{2^{p+l} k(t+1) B\left(\frac{t+p-\frac{k}{2}+2}{2}, \frac{t-p+\frac{k}{2}+1}{2}\right)} {}_2F_1\left(-p, \frac{\frac{k}{2}-t-p}{2}; \frac{t-p+\frac{k}{2}}{2}+1; -1\right). \quad (\text{A28}) \\
\text{e function } {}_2F_1(a, b; c; z) \text{ and } B(\eta, v) = \Gamma(\eta)\Gamma(v)/\Gamma(\eta+v) \text{ are the Gaussian hypergeometric and beta function, respectively, } \Gamma(\dots) \text{ being the gamma function. Then} \\
G_{mm'm'n'}(\chi, l, \mu) &= (-1)^{n+n'} \Xi_m \Xi_{m'} n! \sqrt{(n-|m|)! (n+|m|)! n'!} \sqrt{(n'-|m'|)! (n'+|m'|)!} \\
& \times \sum_{k=0}^{n-|m|} \frac{(-1)^k}{k!(n-k)!(n-|m|-k)!(|m|+k)!} \sum_{k'=0}^{n'-|m'|} \frac{(-1)^{k'}}{k'!(n'-k')!(n'-|m'|-k')!(|m'|+k')!} F_{mm'm'n'}^{(1)}(\chi, l, k, k', \mu), \quad (\text{A29}) \\
F_{mm'm'n'}^{(1)}(\chi, l, k, k', \mu) &= \frac{1}{2^{|\mu|}} \sum_{p=1}^{\mu[\frac{l}{2}]} \xi_p(\mu) \sum_{q=1}^{\infty} 2^q \frac{(l-2p-\chi)(l-2p-\chi+1)\dots(l-2p-\chi+q-1)}{q!} \\
& \times \Omega(2n-2k-|m|+2n'-2k'-|m'|-1+2, 2k+|m|+2k'+|m'|-1+q), \quad (\text{A30})
\end{aligned}$$

Mie Scattering for Spheres $m=1.33$ (water)

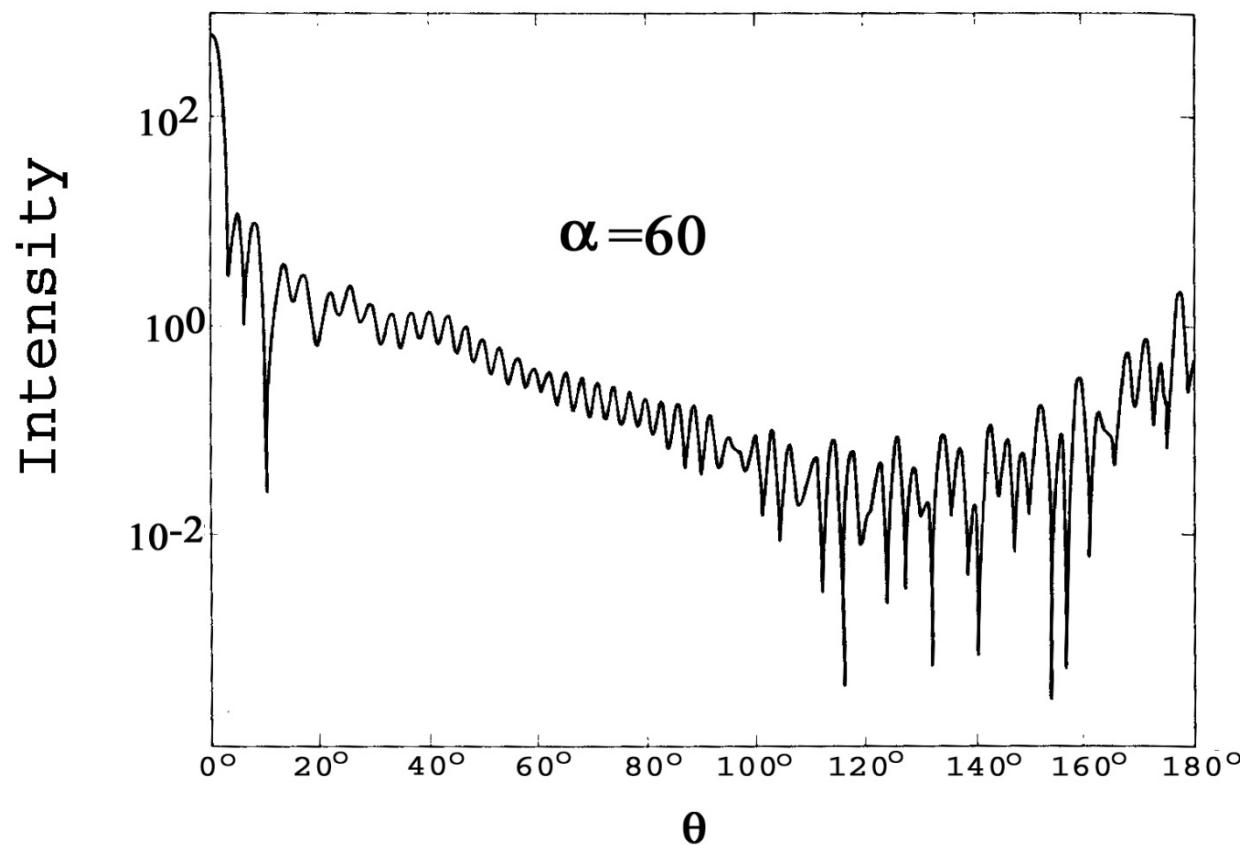
Various size parameters = $2\pi R/\lambda$



Mie Scattering for a Sphere

$m=1.33$

$\alpha = \text{size parameter} = 60$



Beyond Rayleigh scattering...

The scattering wave vector,

q

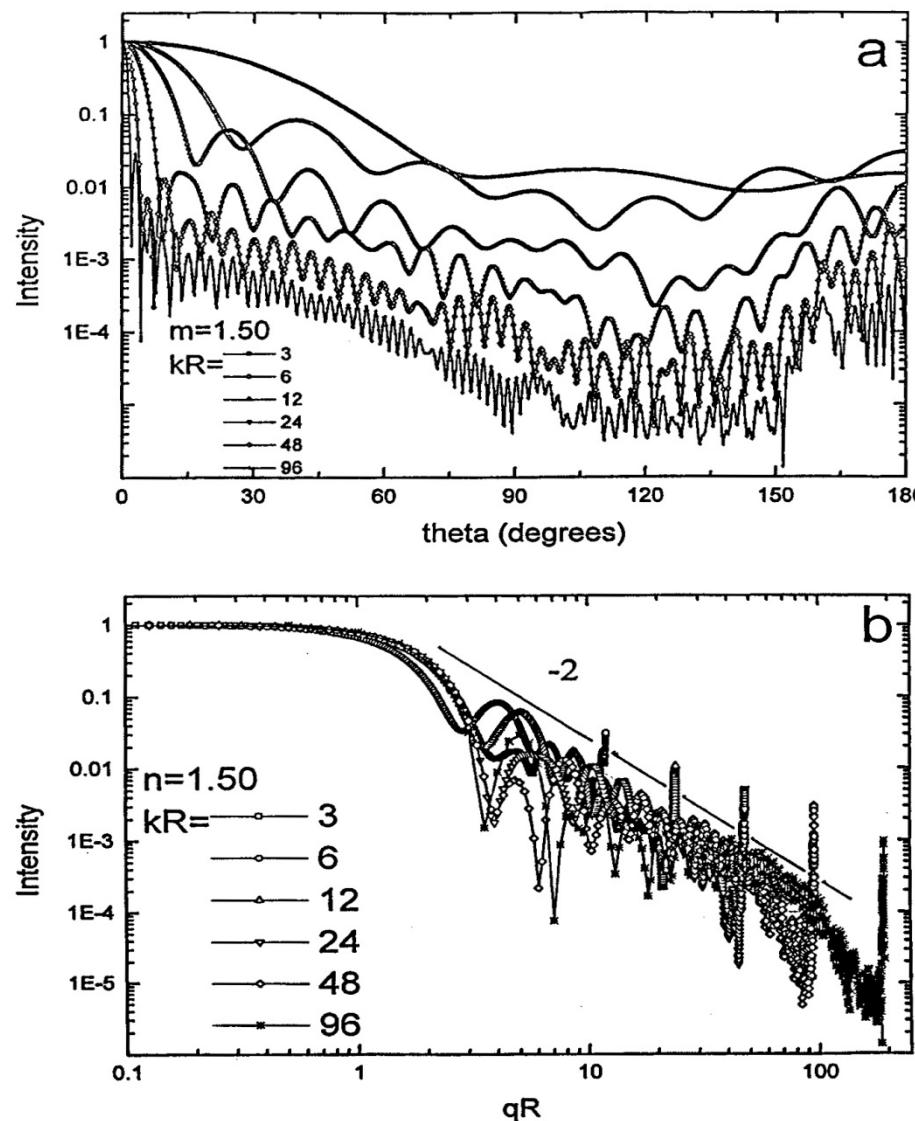
Diffraction theory leads to

$$q = (4\pi/\lambda)\sin(\theta/2)$$

Much more useful than the scattering angle, θ .

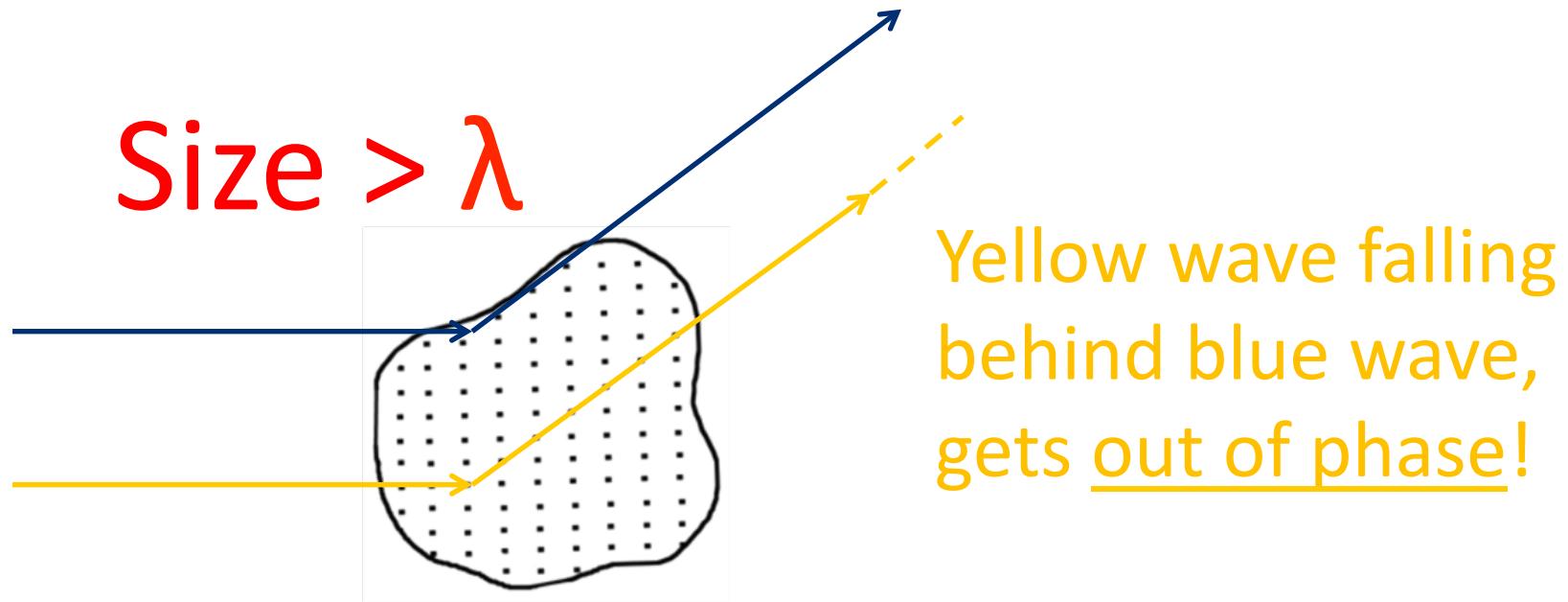
The units of q are (length) $^{-1}$.

Patterns in Mie Scattering



Big Particles

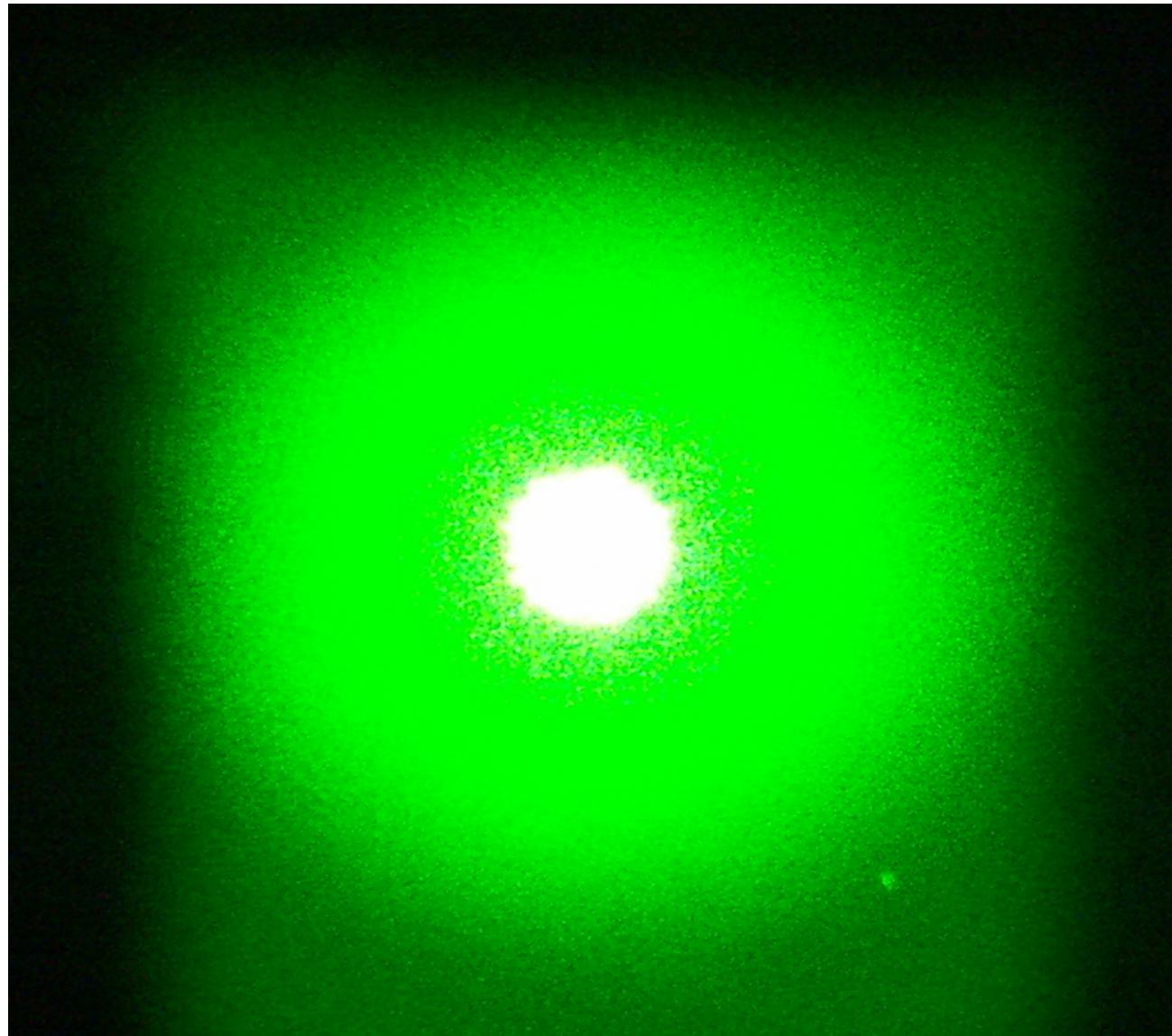
To detector



Waves from different scatterers of the object begin falling out of phase at an angle of

$$\theta \approx \lambda/\text{size} \text{ (radians)}$$

Forward scattering projected on a wall
from 9.6 μ m polystyrene microspheres in water.





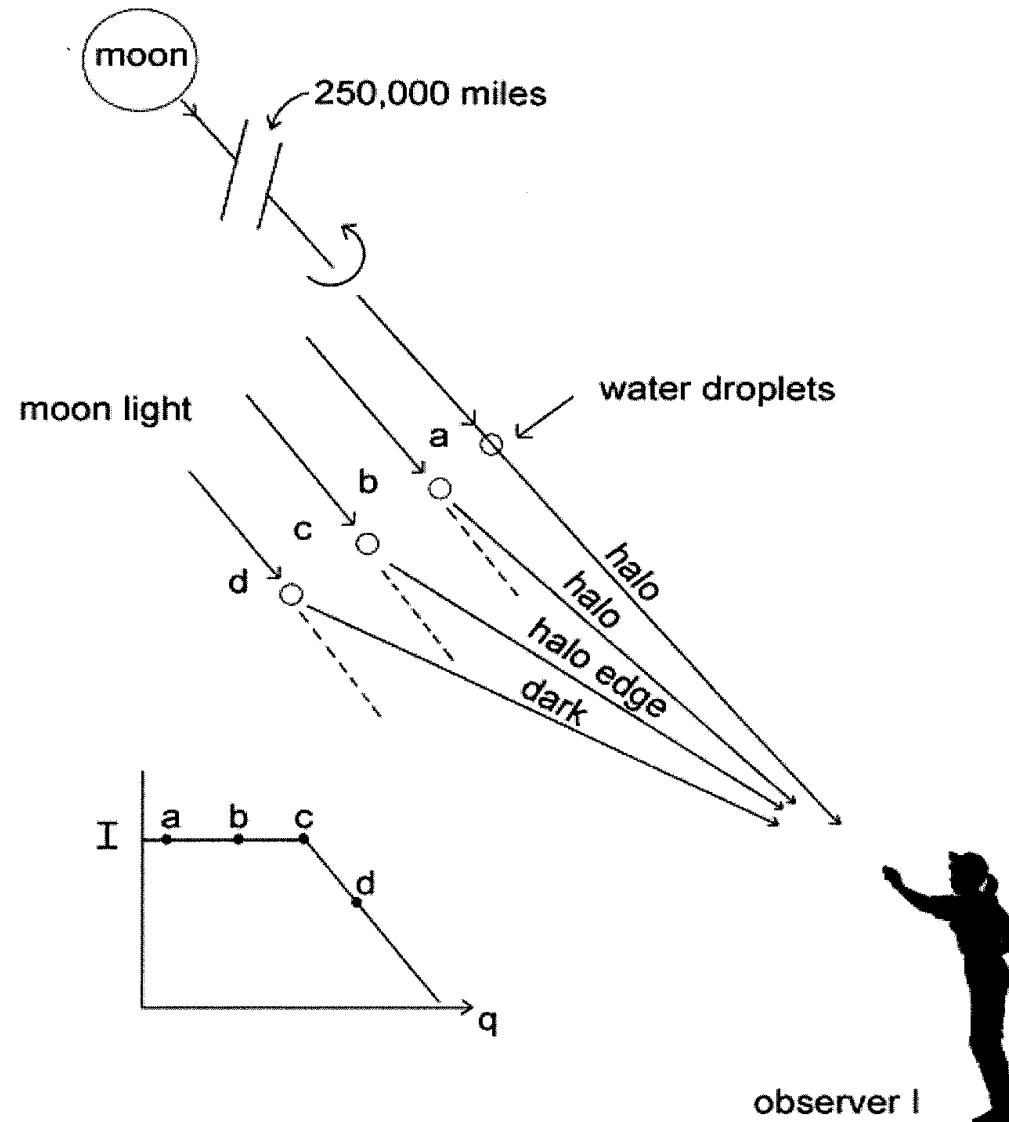
The Lunar Corona



The Lunar corona



Lunar Corona

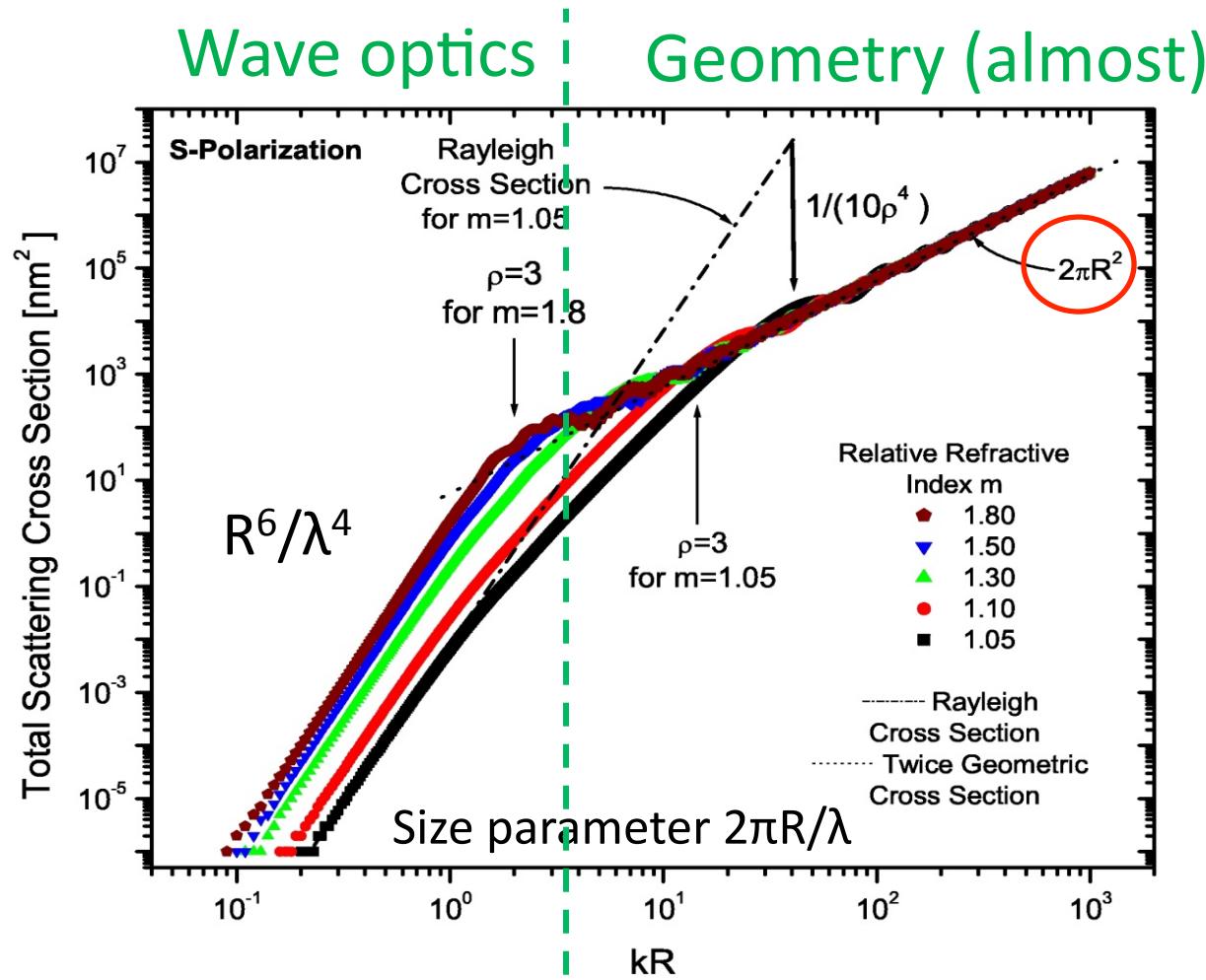


Backlit Cloud

photo.tutsplus.com

Total Scattering Cross Section

Add up all angles



Radius (μm) = 0.01

0.1

1.0

10

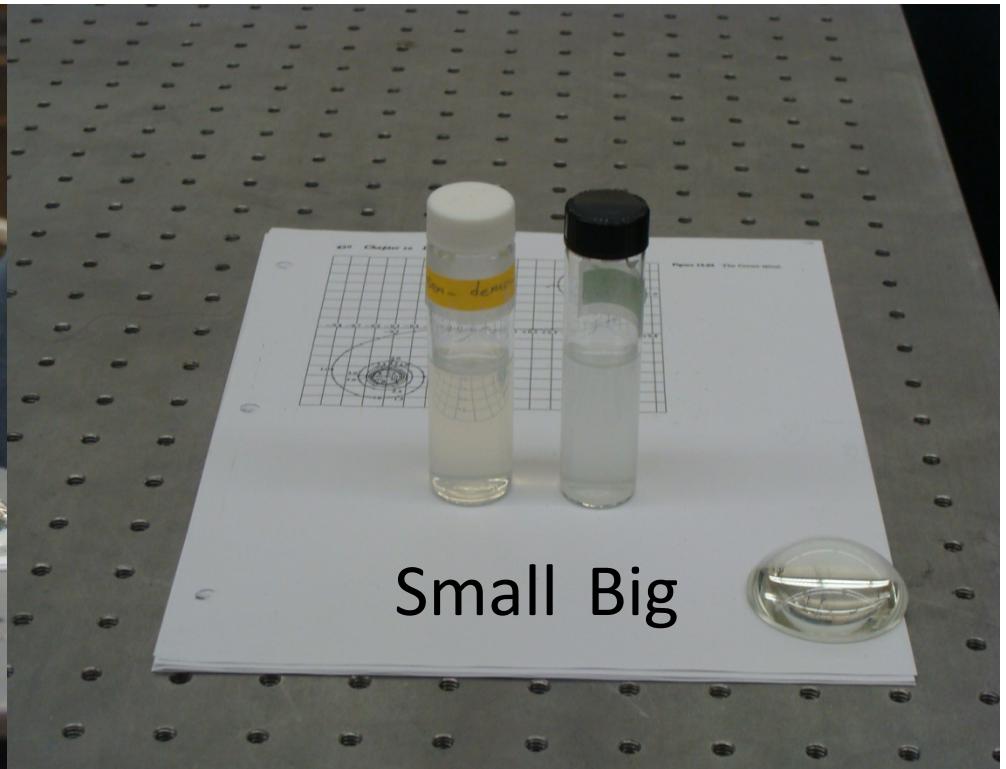
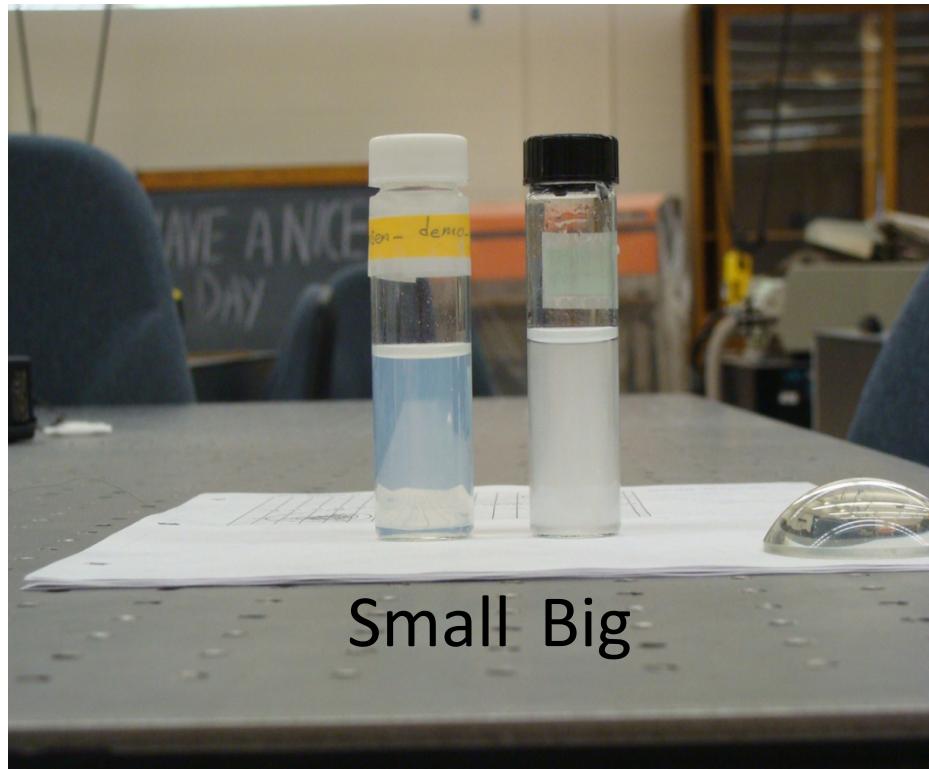
100

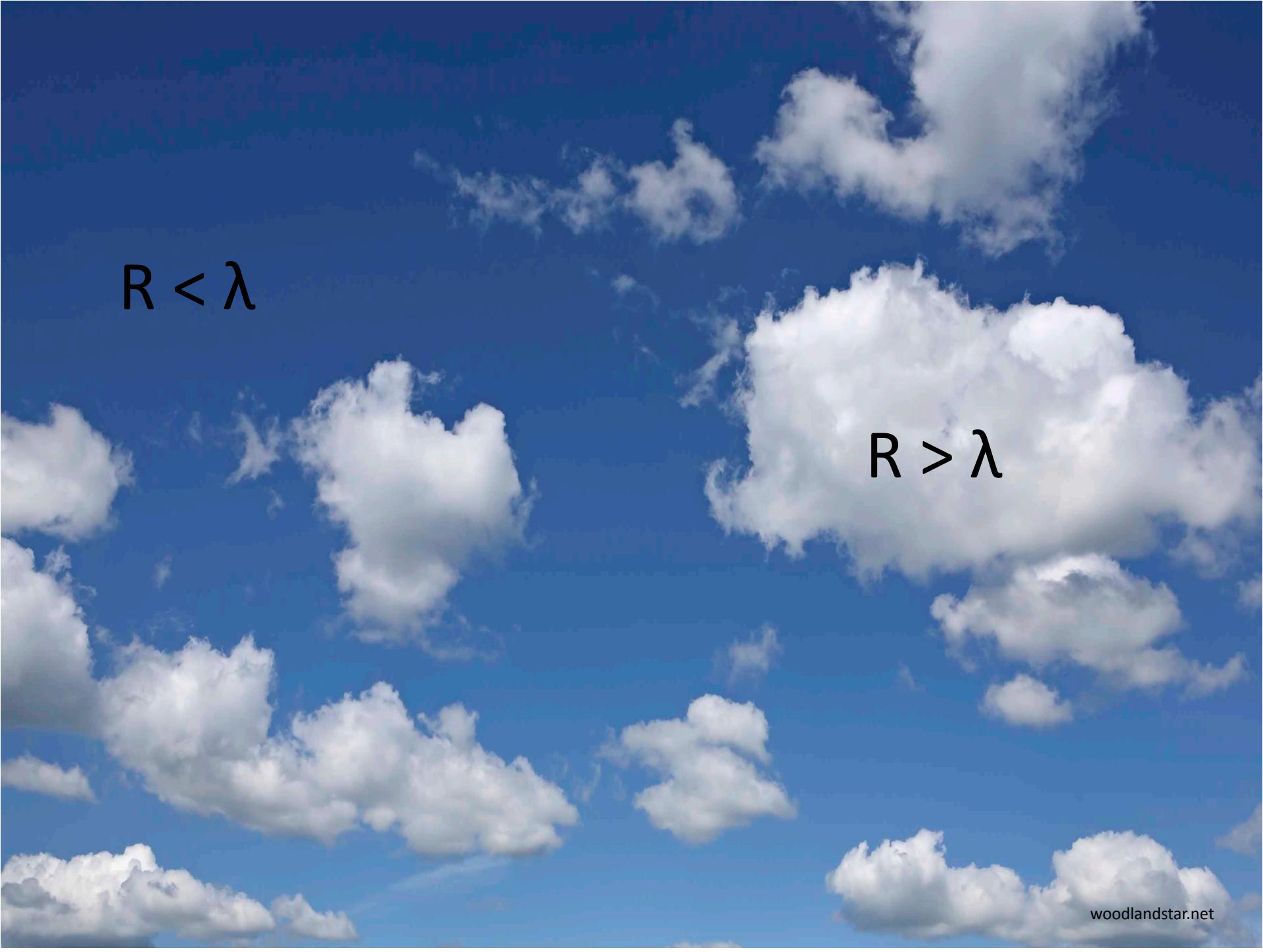
$\lambda \approx 0.5\mu\text{m}$

0.1 mm

Small particles ($R < \lambda$) $C_{sca} \sim R^6/\lambda^4$ Blue scatt, Red trans

Big particles ($R > \lambda$) $C_{sca} \sim R^2$ no λ -dependence, White scatt/trans





$R < \lambda$

$R > \lambda$

Comet Hale-Bopp

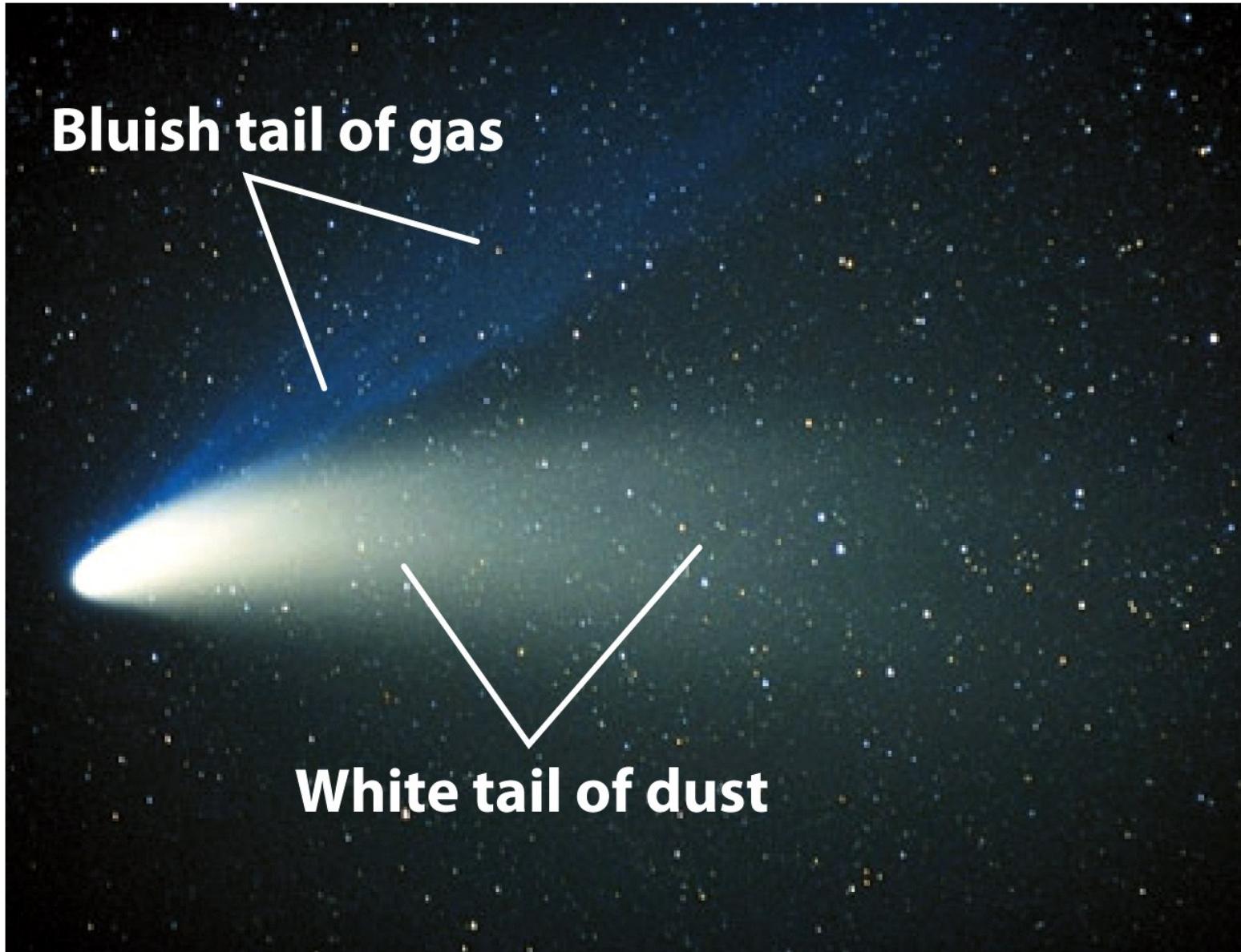


Figure 4-8
Investigating Astronomy
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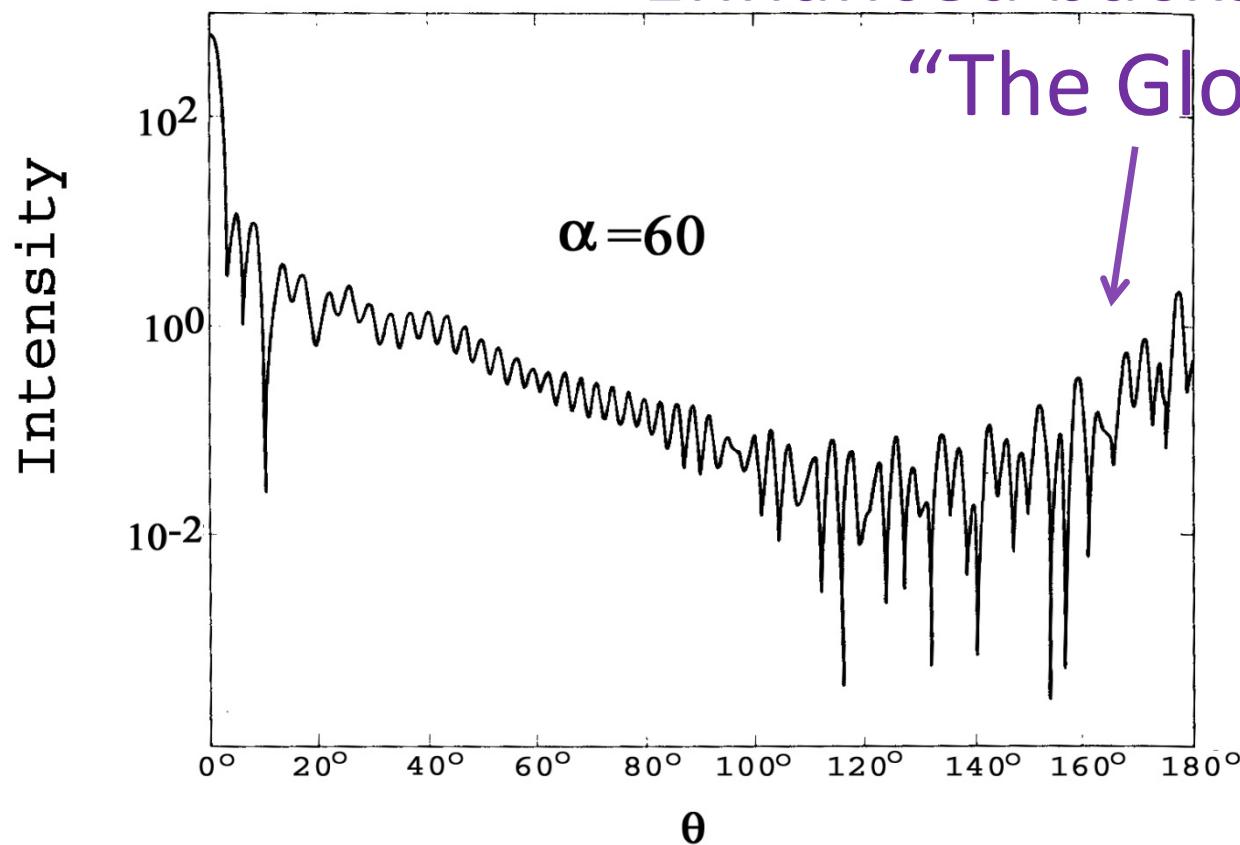
Mie Scattering for a Sphere

$m=1.33$ ([water](#))

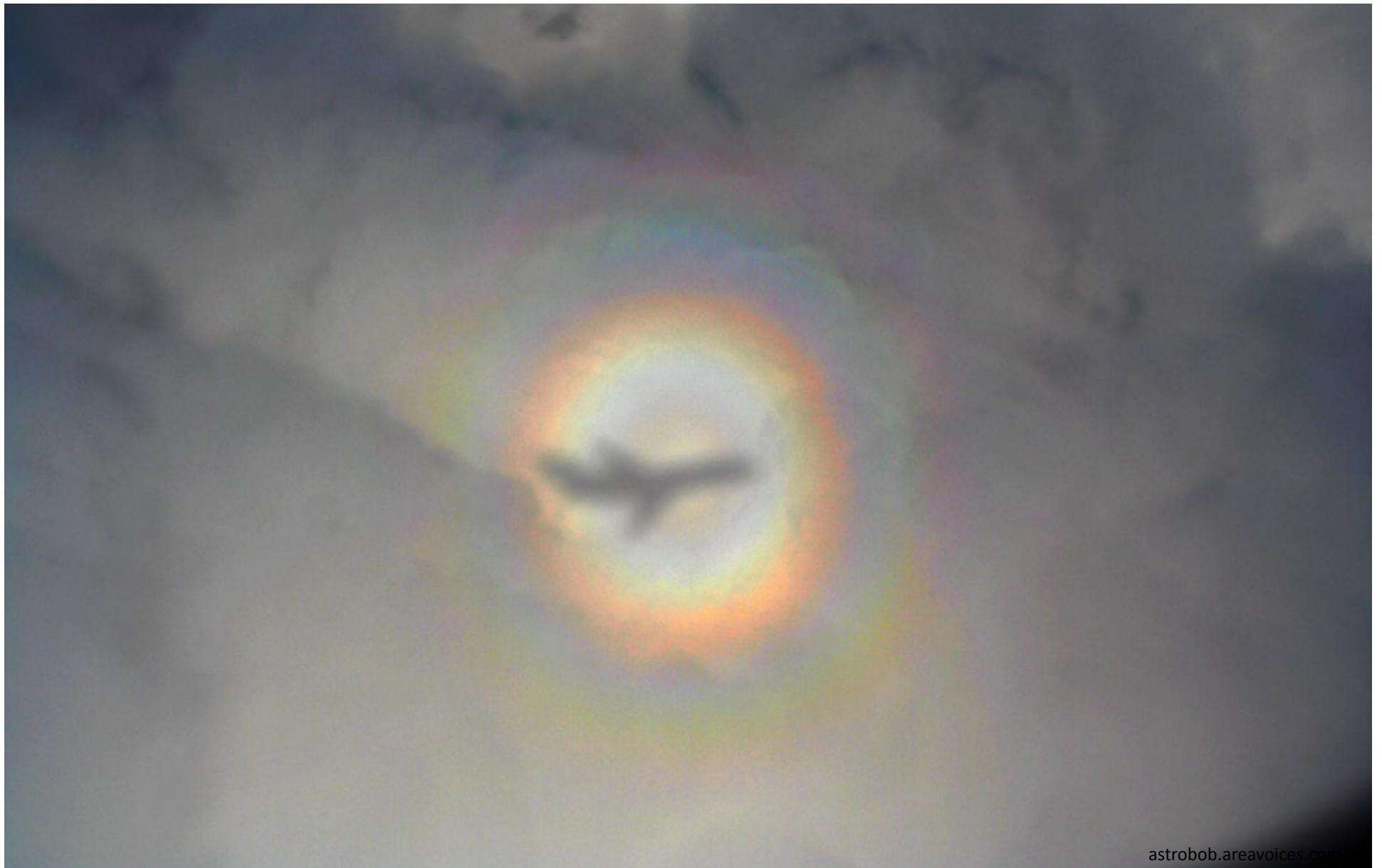
α = size parameter =60

Enhanced backscattering,

“The Glory”



The Glory around an airplane's shadow



Front Lawn Heilgenshein



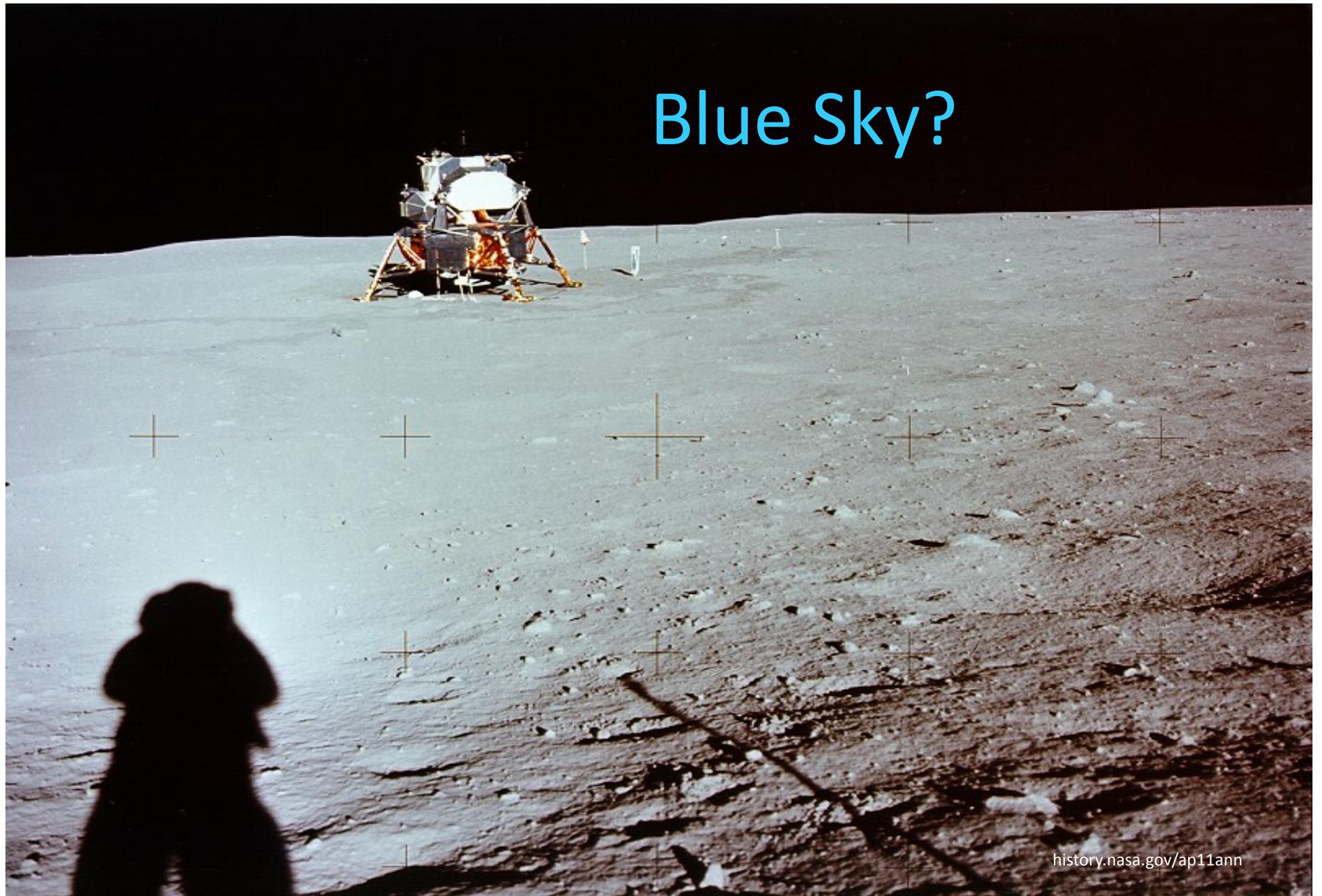
B J Ackerson Oklahoma state Univ.

The False Glory The Heilingenschein



Note glow around the photographer's shadow.

Blue Sky?



Z3



KANSAS OCT
- 20AQM



Heilingenschein





Pollution



earthtimes.org/health/



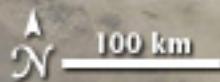
Desert Dust

•Beijing

Bohai Sea

Taihang Shan

news.discovery.com

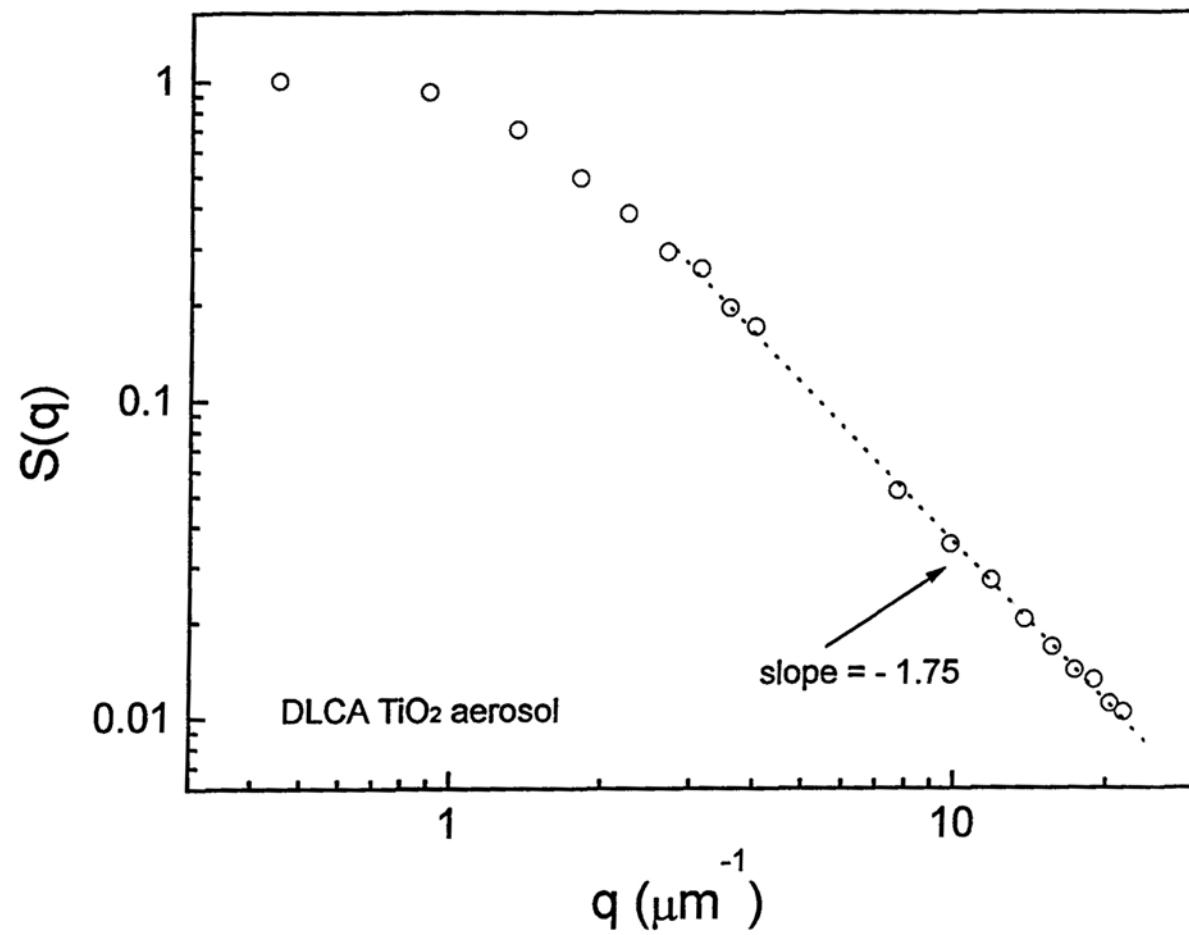


Fractal Aggregates (soot)

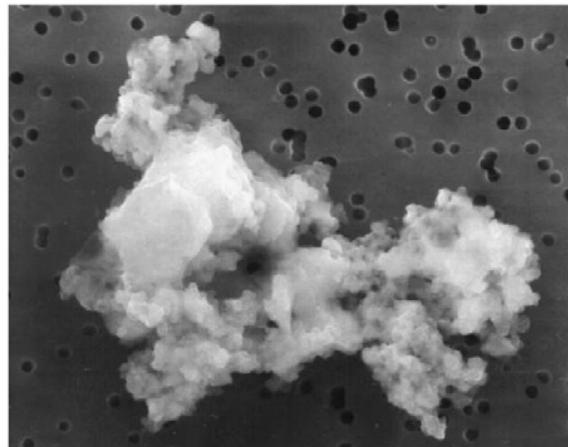
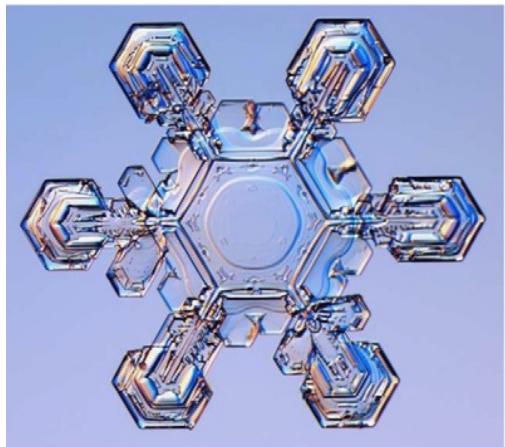
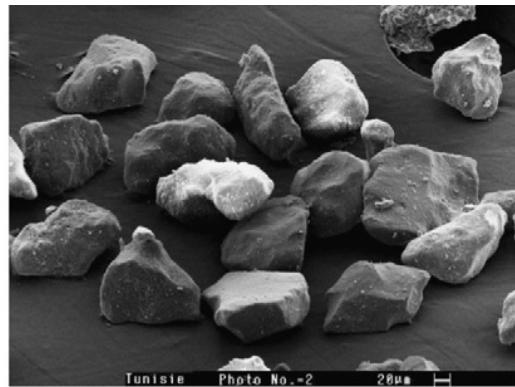
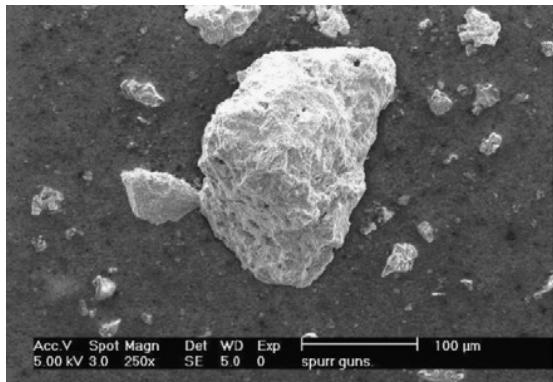


Aerosol Sci. Tech. 35, 648 (2001).

Scattering from fractal aggregates

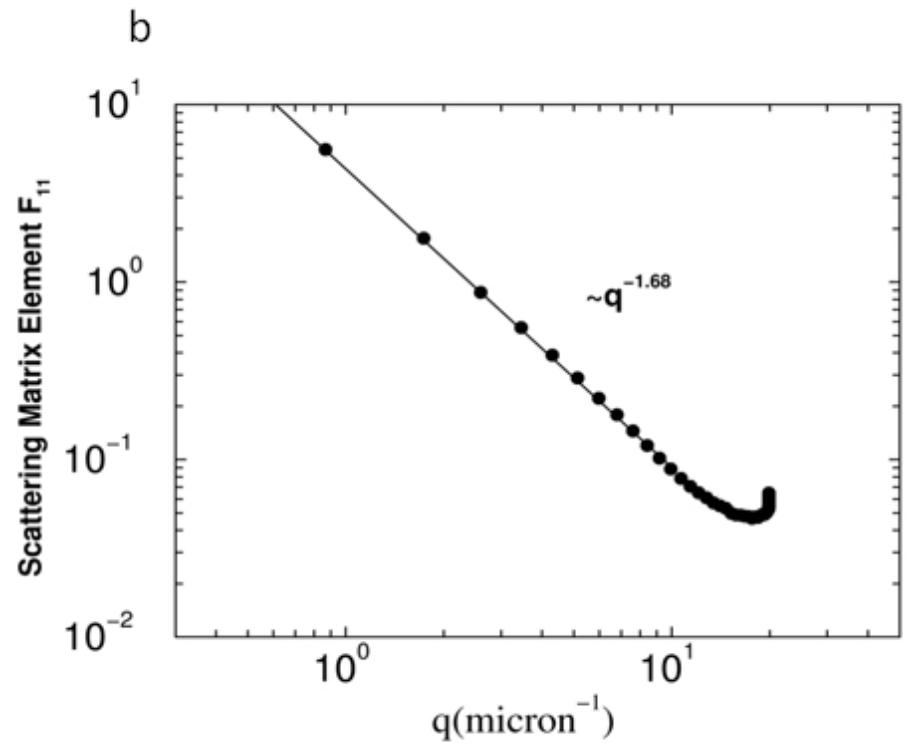
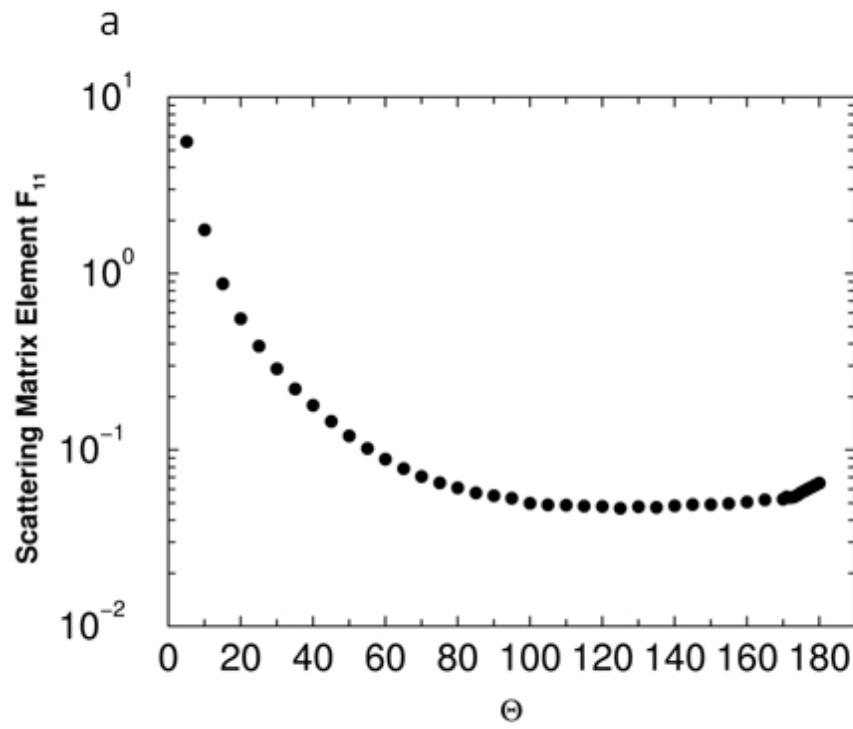


Particle Shapes



Scanning electron microscope images of volcanic ash from Mount Spurr volcano (top left), desert dust from Niger (top right) and an interplanetary dust particle collected at high altitude in the atmosphere of the Earth (bottom right). The white bars denote 100, 20, and 1 μm , respectively. The bottom left shows a snowflake. From Munoz & Hovenier JQSRT 112 (2011) 1646–1657.

Scattering from Desert Dust

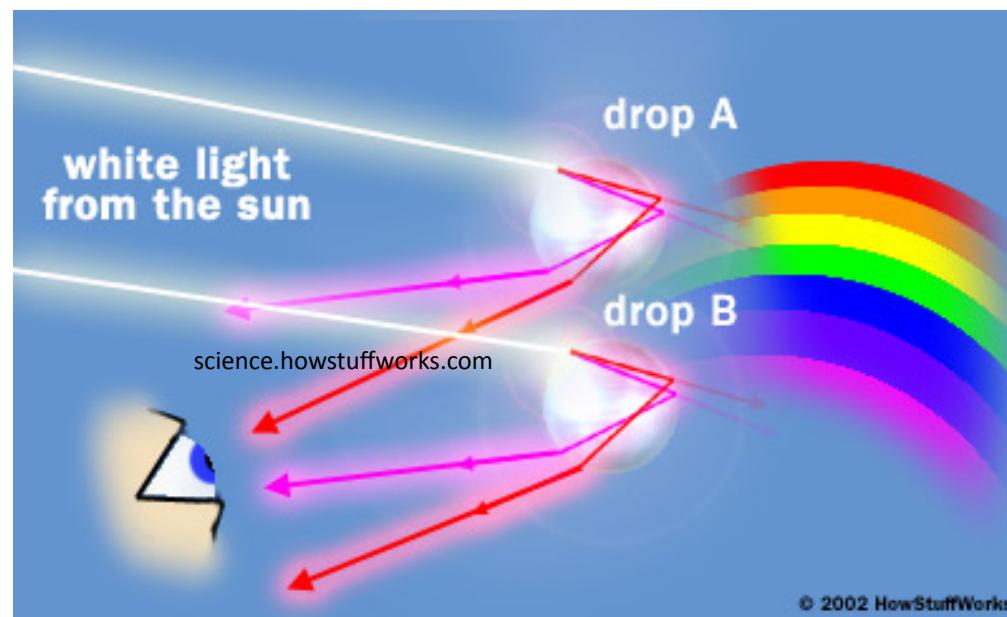
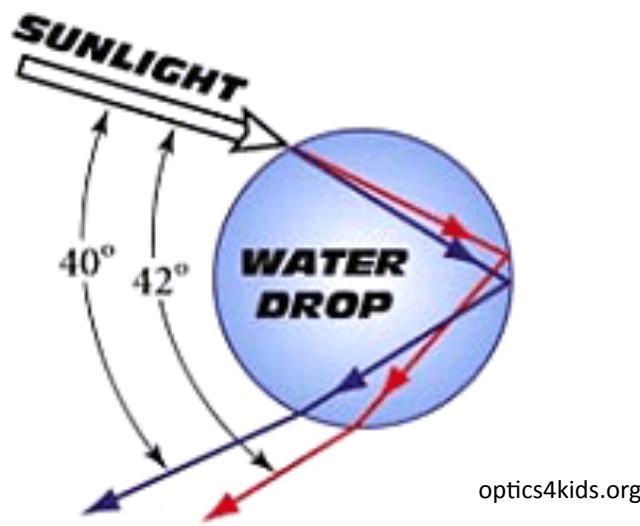


Power Laws!

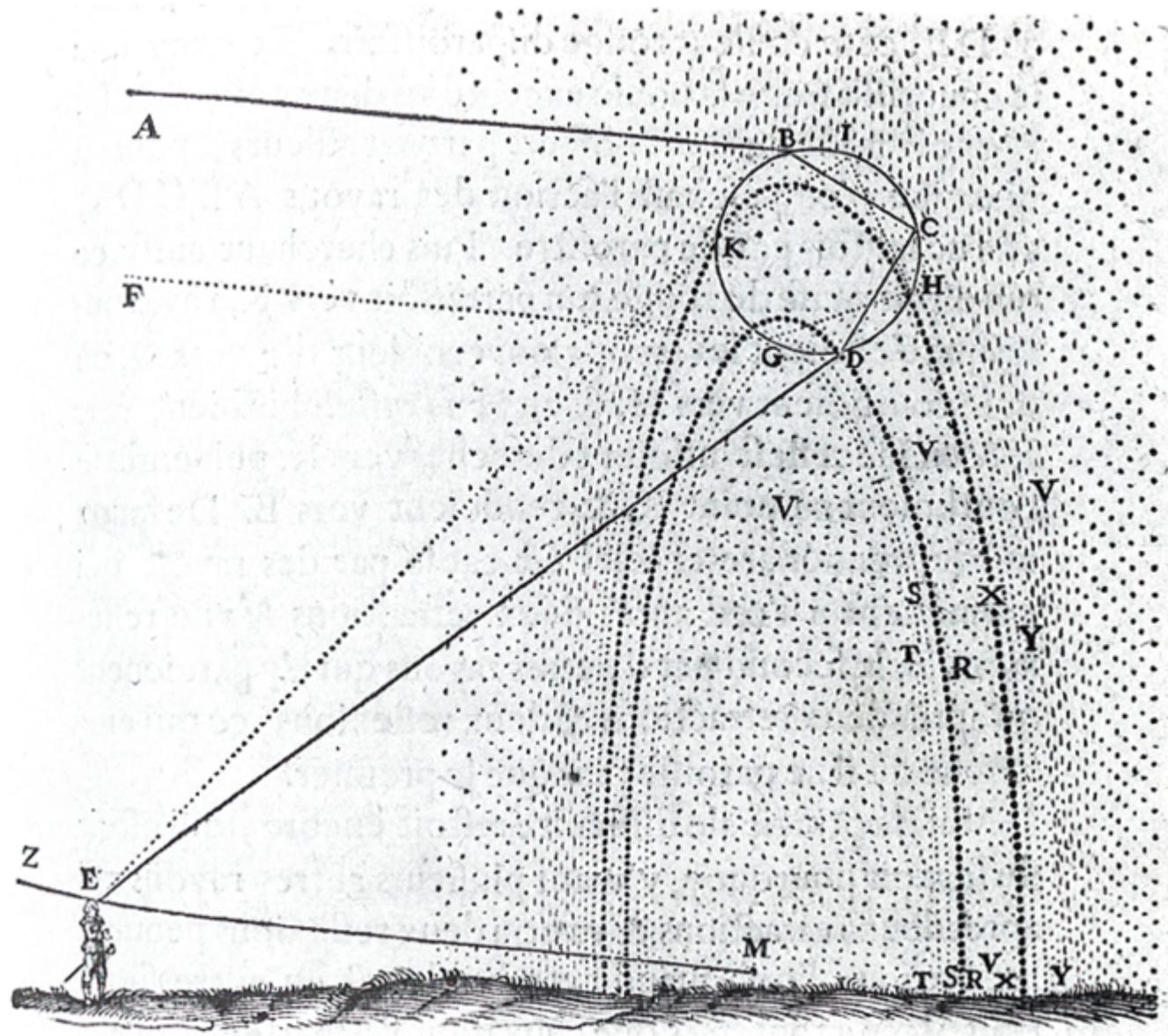
The Rainbow



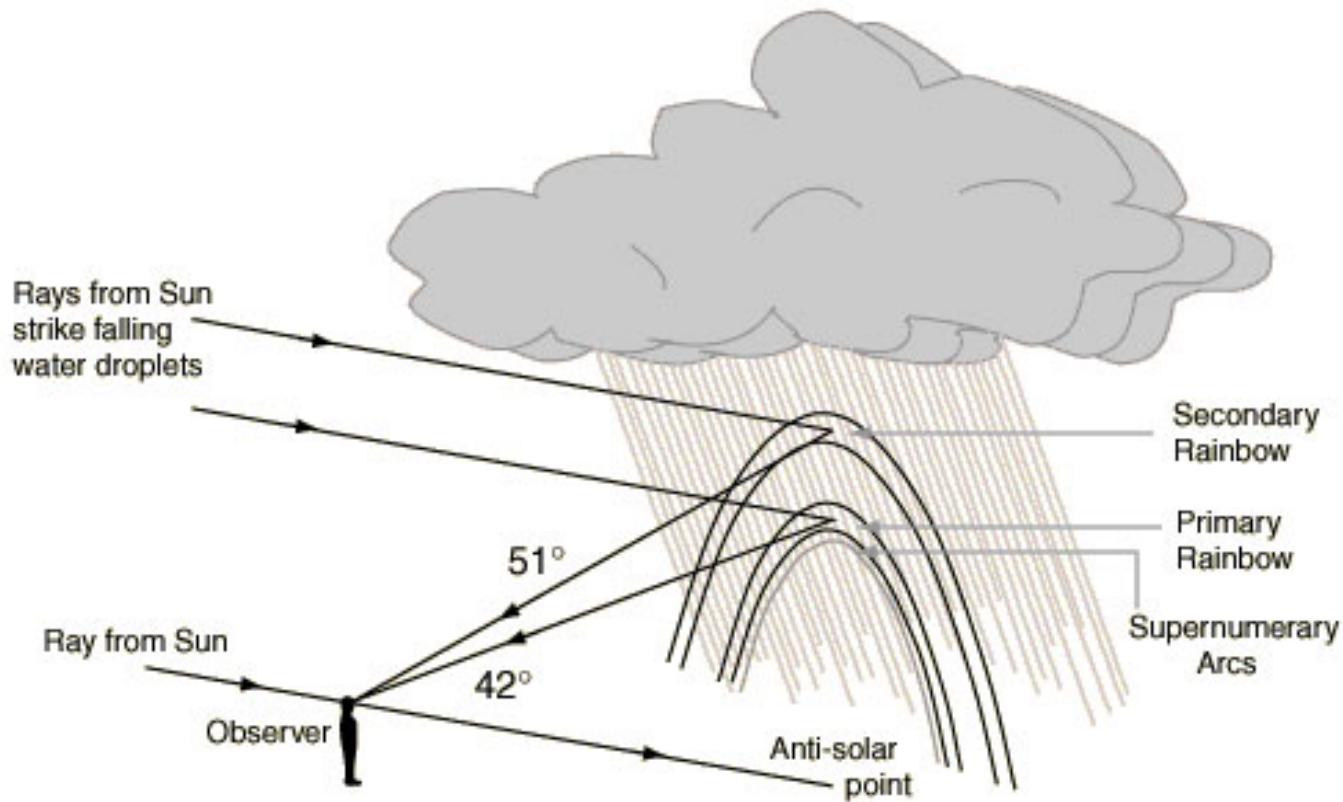
Rainbow Optics



Descartes's Explanation of the Rainbow (1656)



Rainbows form the arc of a perfect circle centered on the shadow of your head.



Rainbow and Anti-solar Point



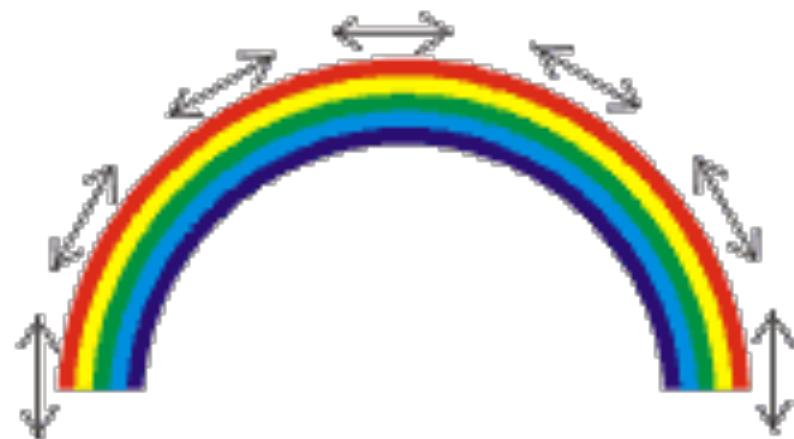
wordlesstech.com

Rainbow and Anti-solar Point



X

Rainbow Polarization

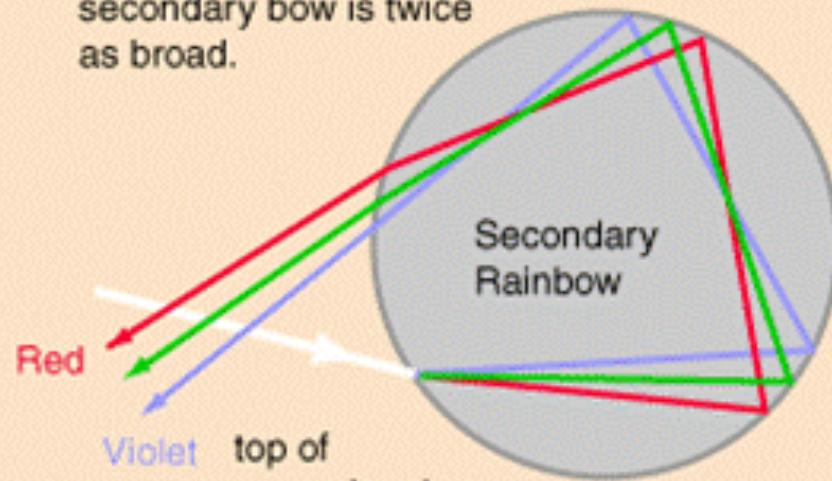


The Double Rainbow

Nick Suydam © 2012
www.nicksuydam.com

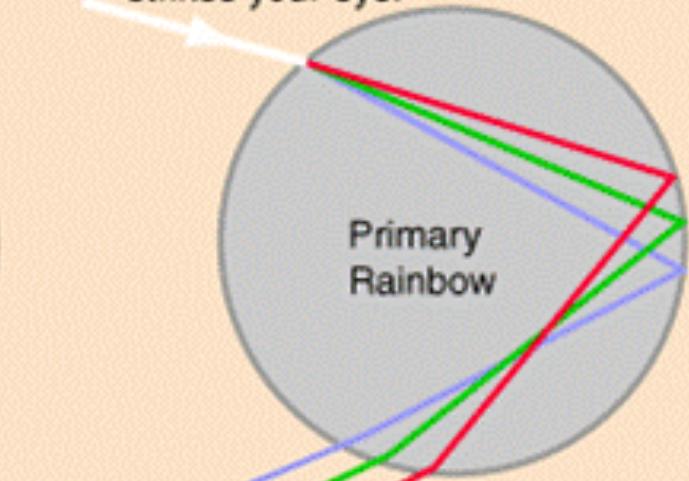
Rainbow Light Paths

The colors of the secondary rainbow are reversed from the primary bow, and the secondary bow is twice as broad.



Violet
Red
top of
secondary bow
since it comes
to the eye from
higher drops.

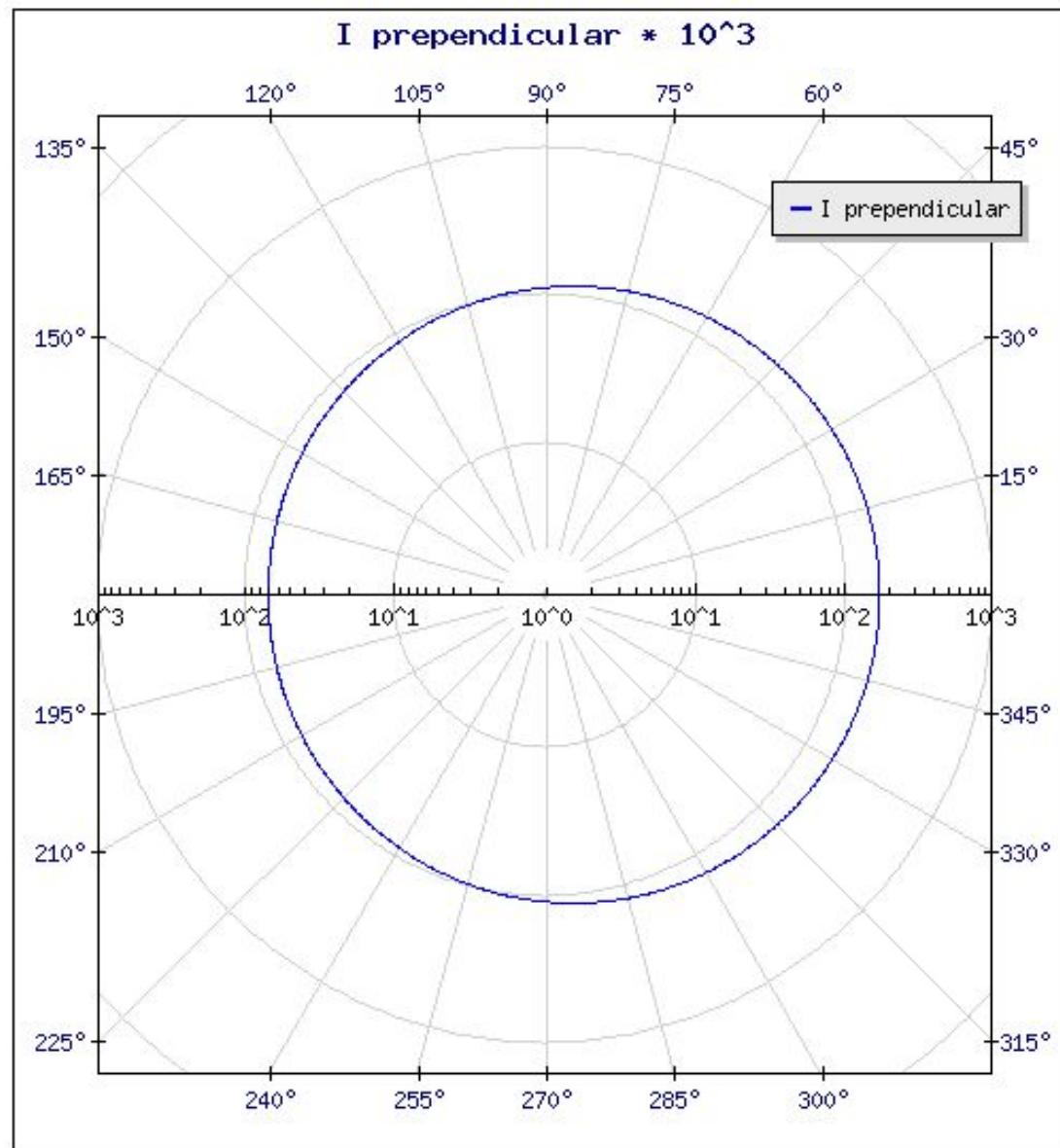
Violet light is bent more and comes out higher from the droplet. It appears at the bottom of the rainbow since violet light from lower droplets strikes your eye.



Violet
40°
42° Red
The red light from
droplets higher in the sky
reaches your eye.

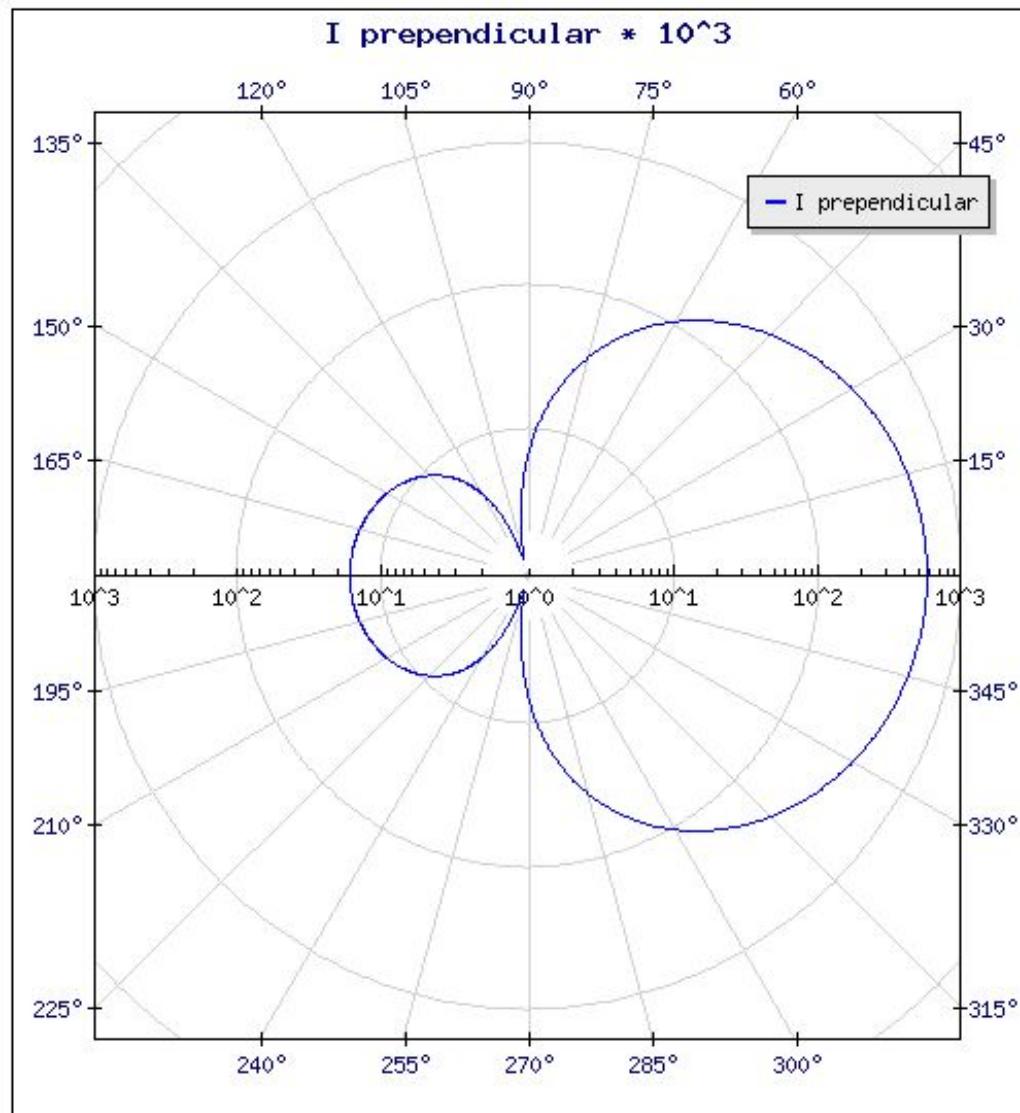
Mie Scattering. Polar plot– log scale

R = 0.2 μm



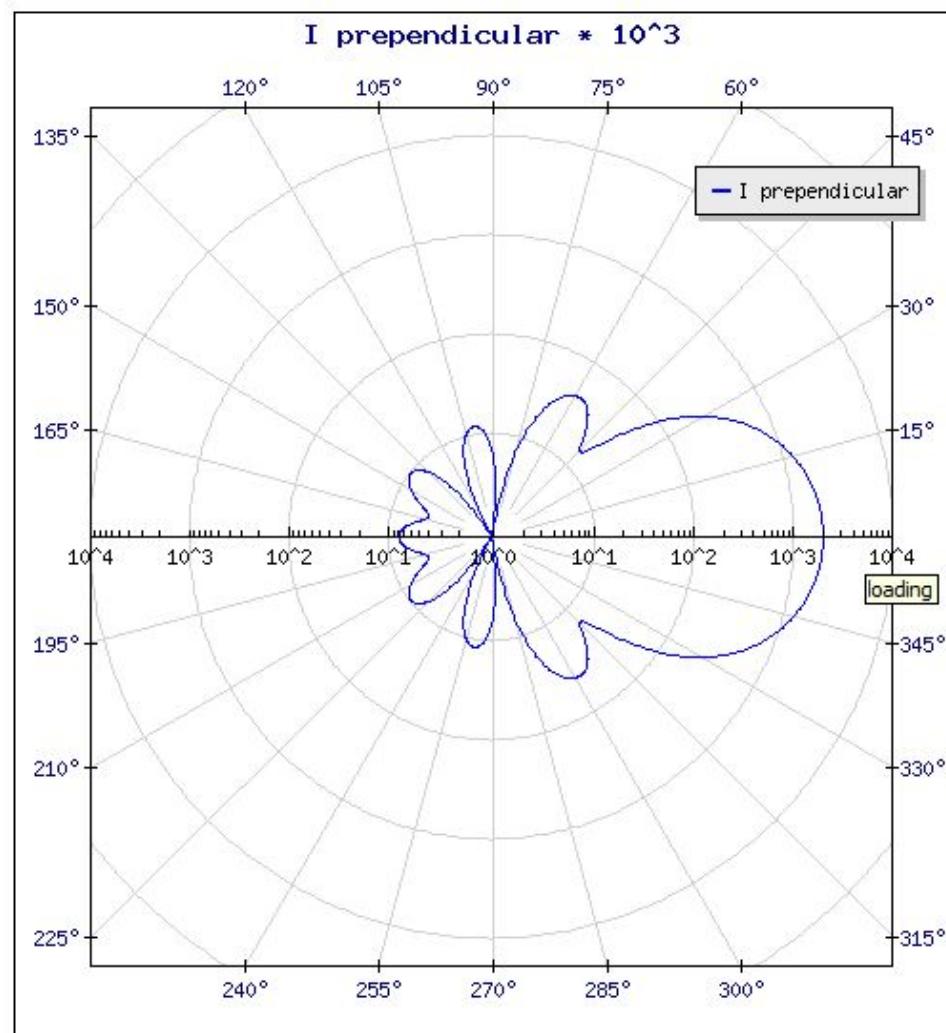
Mie Scattering. Polar plot– log scale

R = 0.5 μm



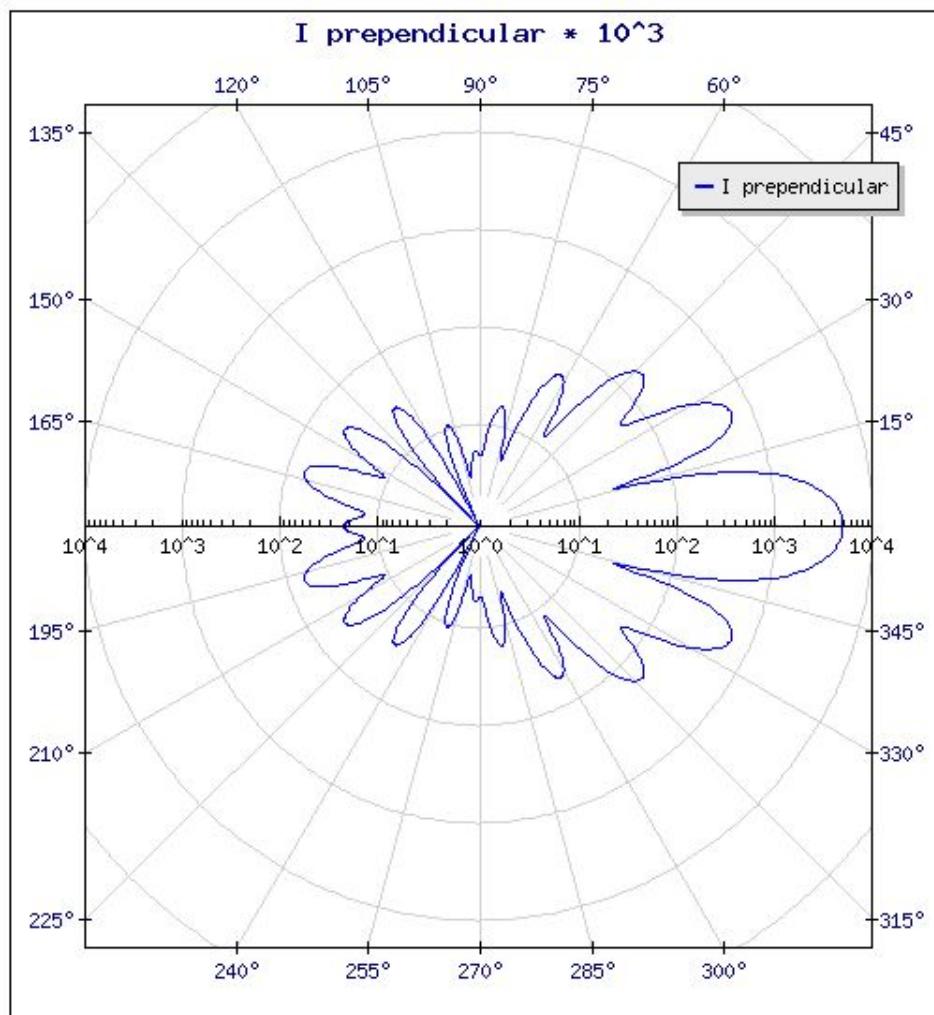
Mie Scattering. Polar plot– log scale

R = 1.0 μm



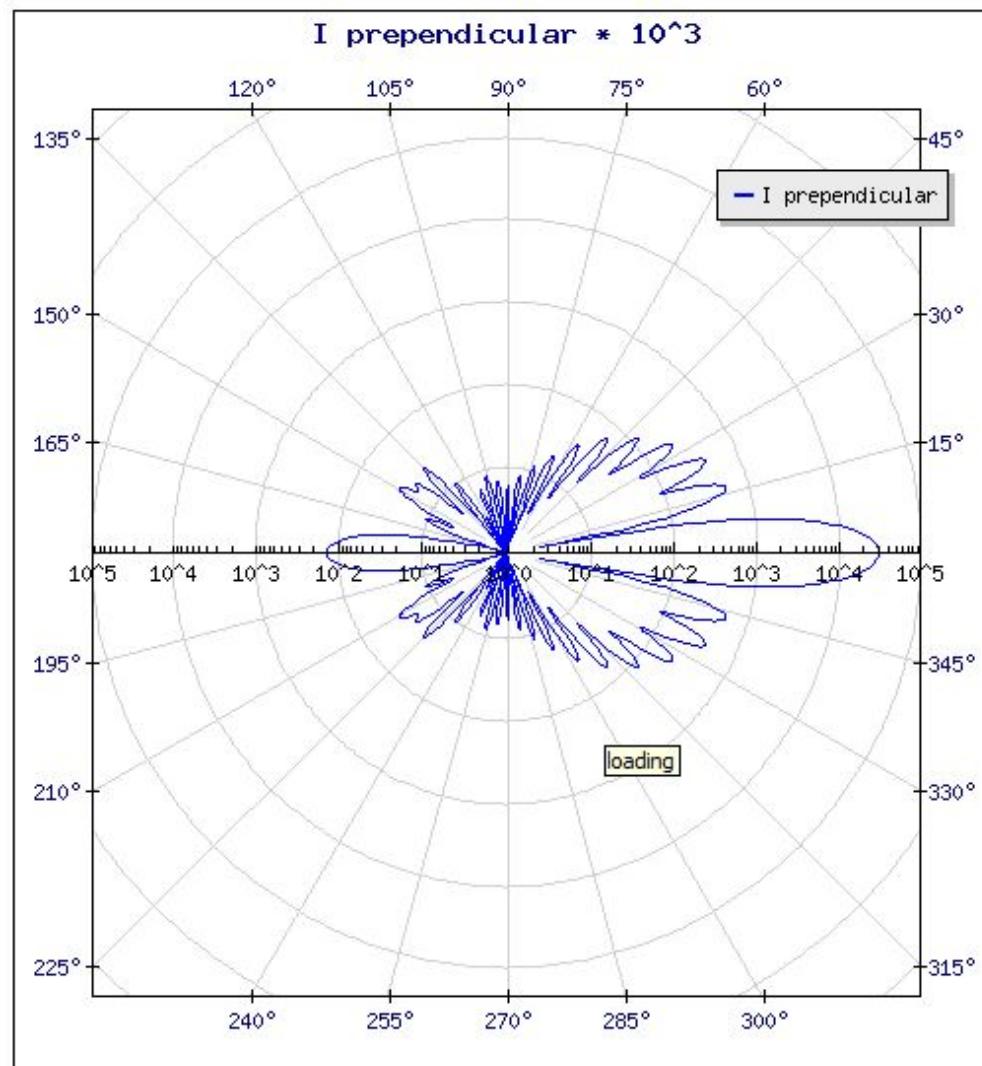
Mie Scattering. Polar plot– log scale

R = 2.0 μm



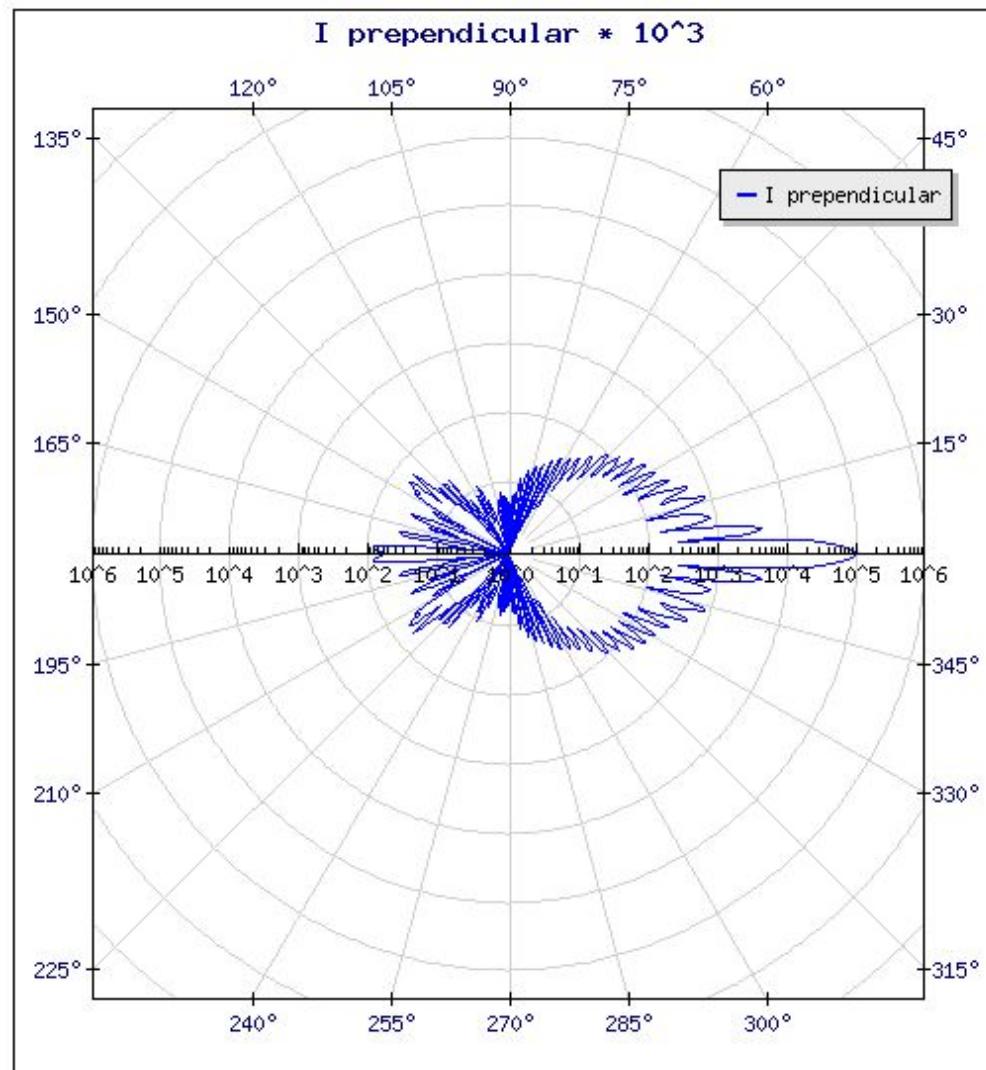
Mie Scattering. Polar plot– log scale

R = 5.0 μm



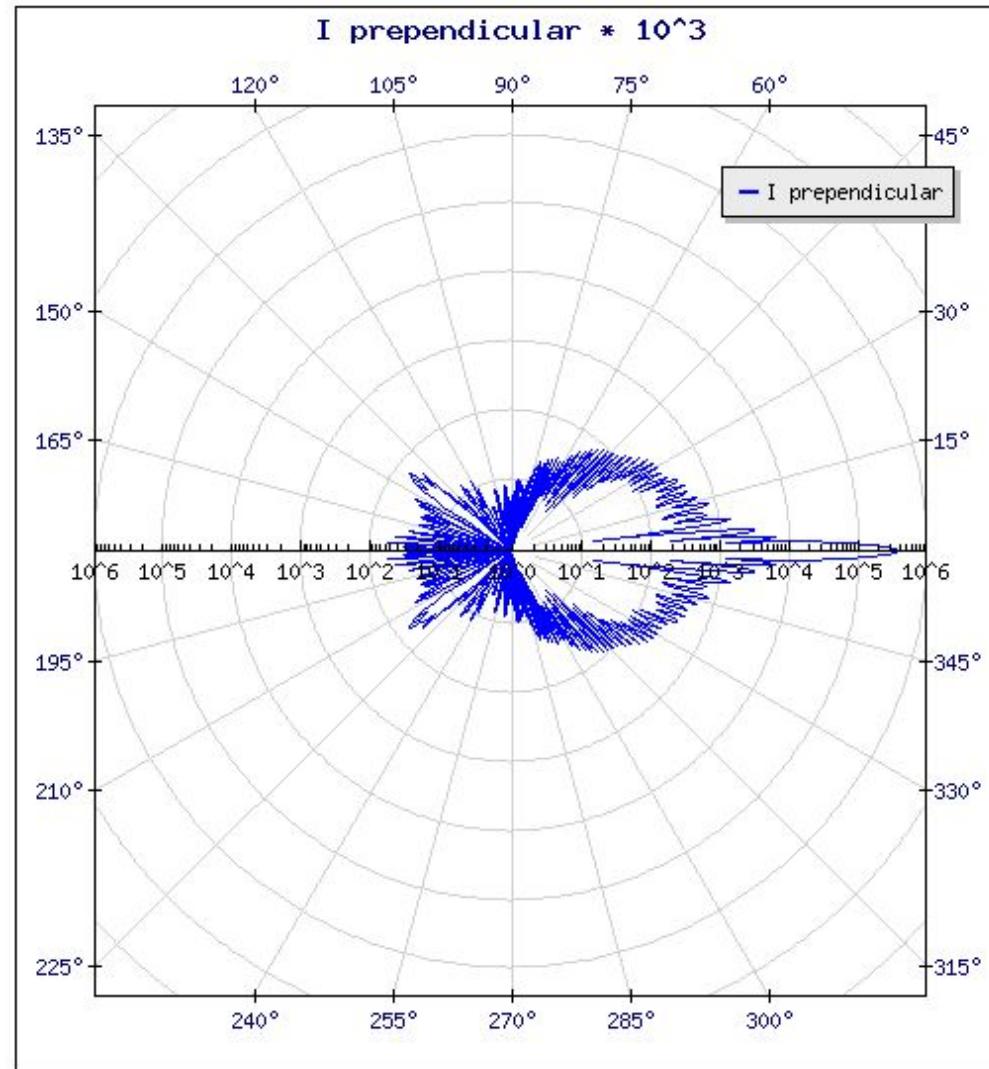
Mie Scattering. Polar plot– log scale

R = 10.0 μm



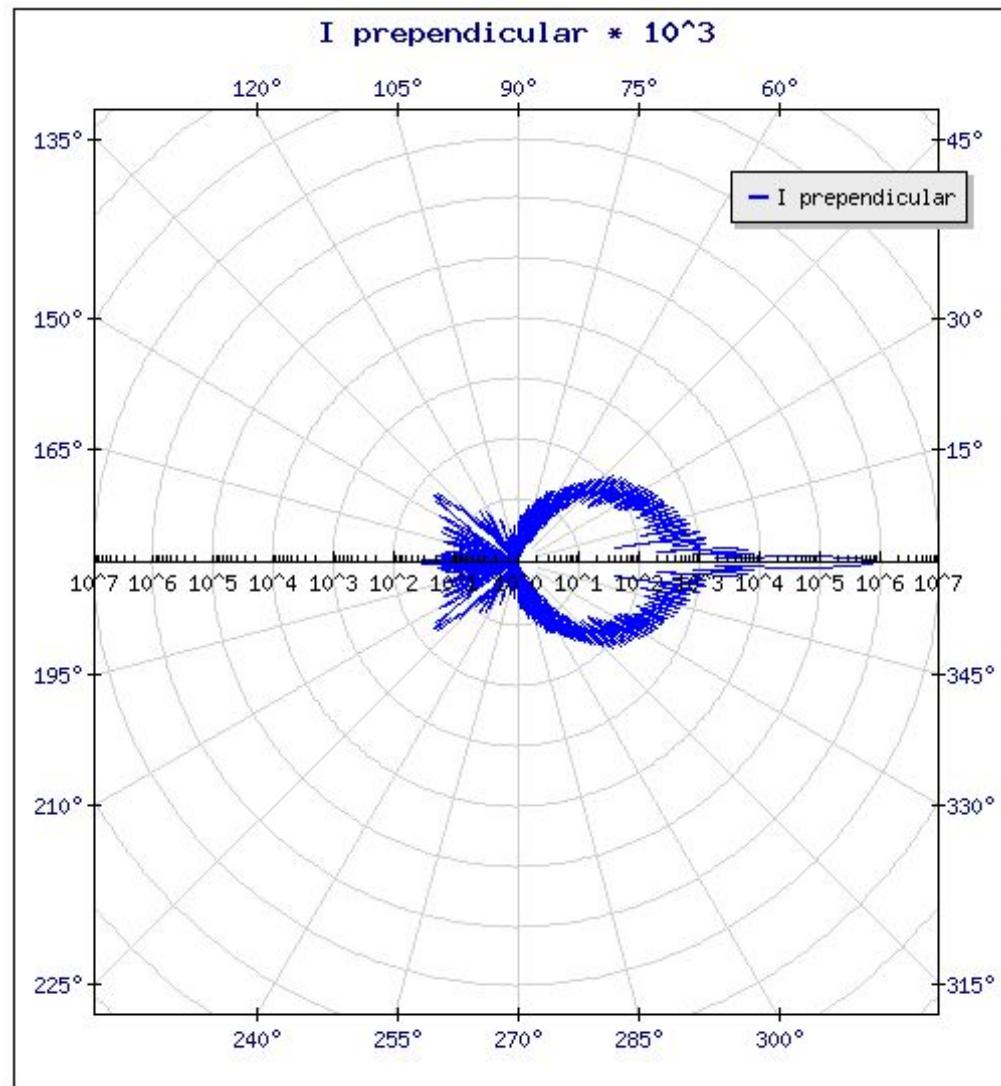
Mie Scattering. Polar plot – log scale

R = 20.0 μm



Mie Scattering. Polar plot– log scale

R = 30.0 μm



Fabiano B. Diniz

Solar Halo



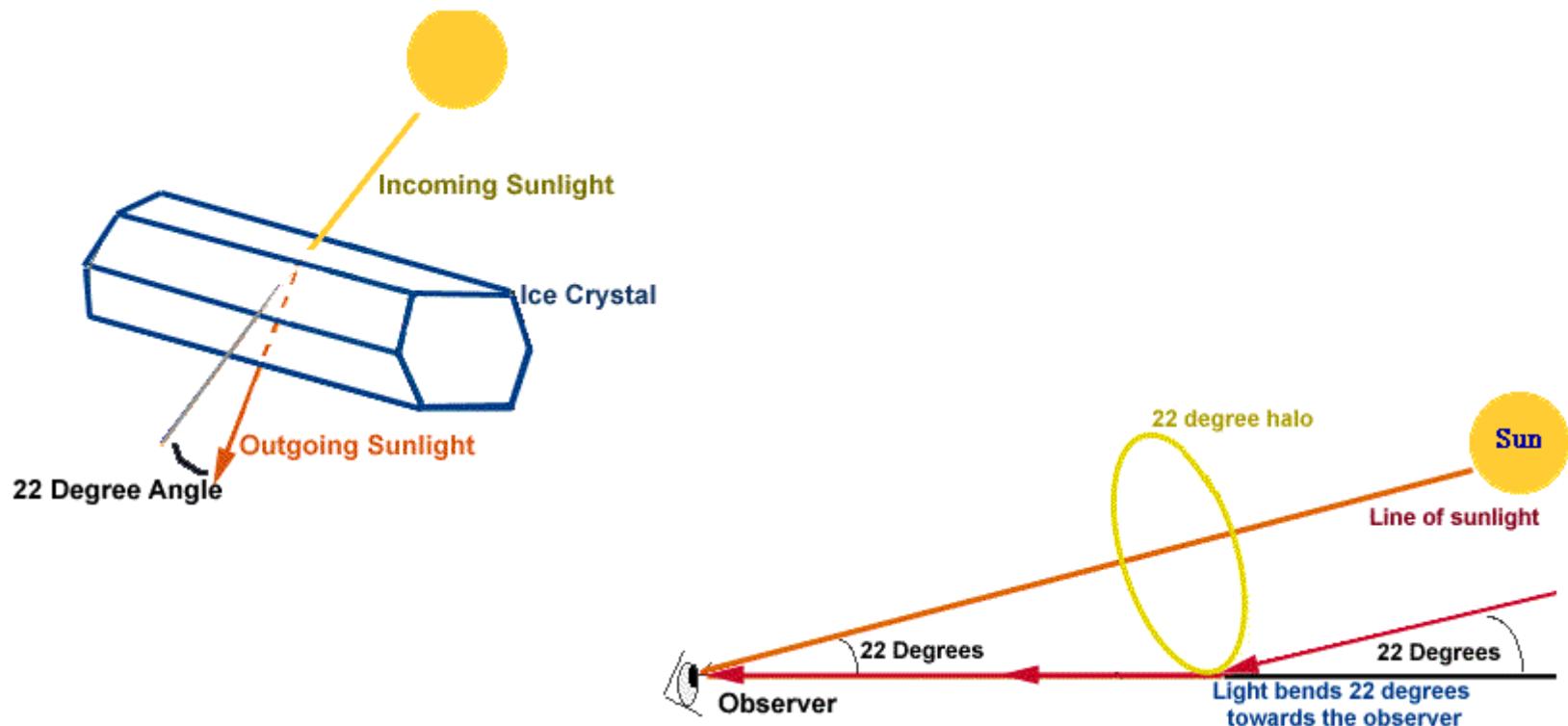
Lunar Halo



Halos

Geometric Optics

Hexagonal Ice Crystals



Sun Dogs



weather.blogs.wkbt.com

Sun Dog



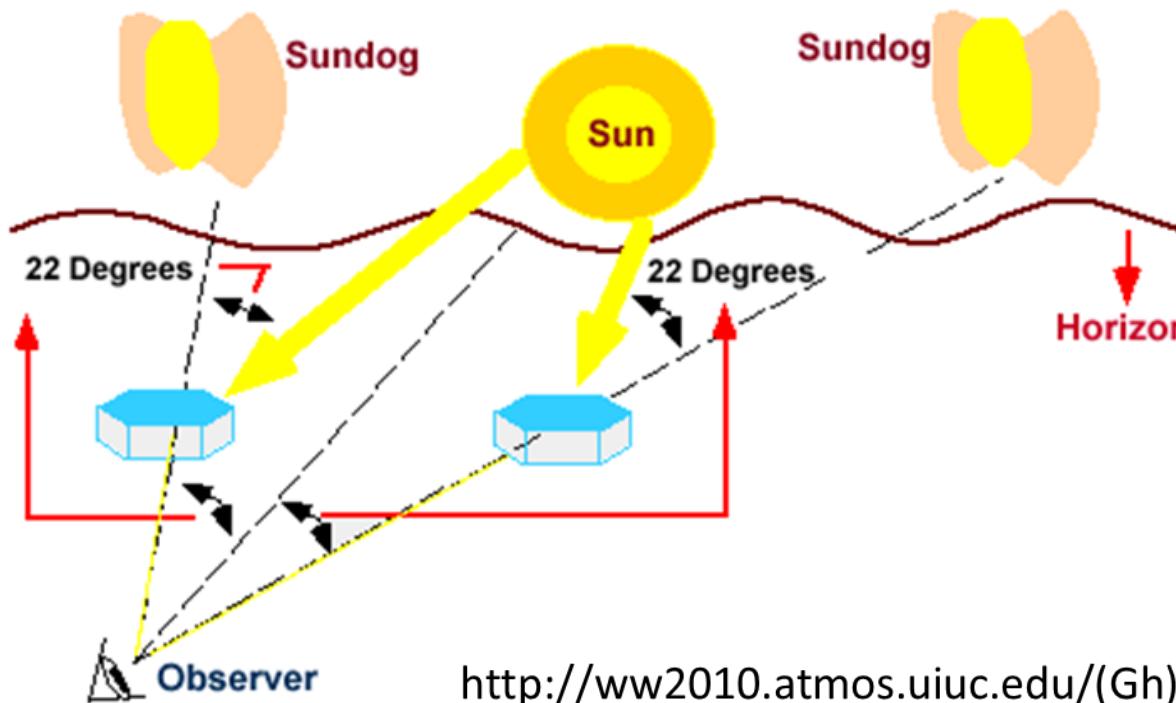
Sun Dog



Sun Dogs

Geometric Optics

Hexagonal Ice Crystals



[http://ww2010.atmos.uiuc.edu/\(Gh\)/guides/mtr/opt/ice/sd.rxml](http://ww2010.atmos.uiuc.edu/(Gh)/guides/mtr/opt/ice/sd.rxml)

And etc. ...

See Minnaert , *The Nature of Light and Color in the Open Air*, Dover



http://nsidc.org/arcticmet/basics/phenomena/sun_dog.html

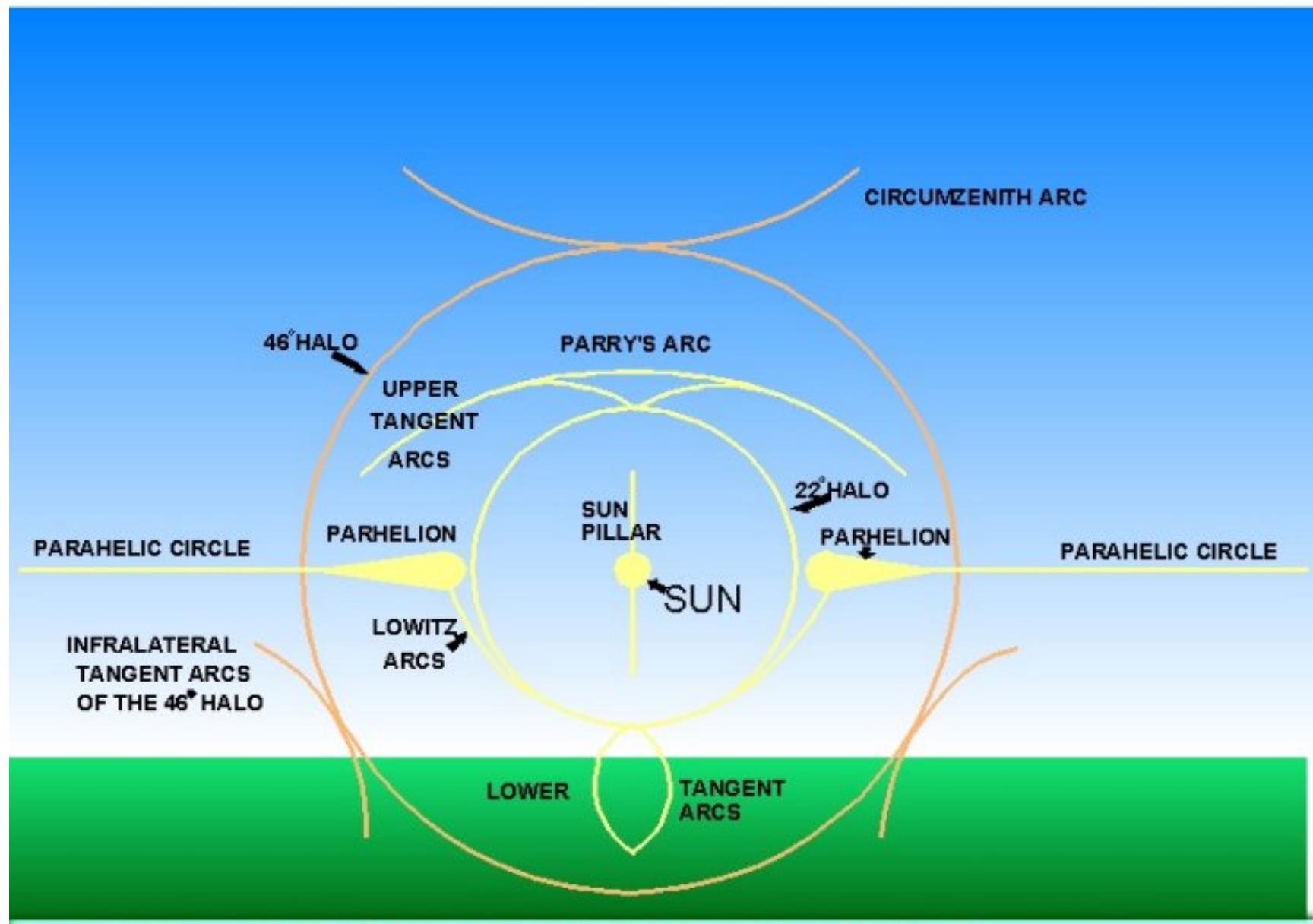


Figure 13-5 Composite of Those Halo Phenomena Which Have Been Observed and Named

Thank You



A scenic landscape featuring rolling green hills under a clear blue sky with scattered white clouds. In the foreground, there are several dead, leafless trees standing in a grassy field. A simple wire fence with wooden posts runs across the lower left corner. The overall scene is peaceful and rural.

Thank you



Thank you

[.artistsagainstwindfarms.com/phil-epp.html](http://artistsagainstwindfarms.com/phil-epp.html)

Consequences of Rayleigh Scattering (1)

Blue sky and red sunset.

$$\sigma = 24\pi^3 V^2 / \lambda^4 \left| \left(m^2 - 1 \right) / \left(m^2 + 2 \right) \right|^2$$

